The marginal efficiency of active search

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Background

- Two types of non-employed workers willing to accept a job (BLS)
  - Passive searchers: e.g., waits for an employer to contact them
  - Active searchers: e.g., contacts an employer about a position

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  - Marginal efficiency of active and passive search are fixed
  - Constant micro-elasticity of unemployment w/r.t. UI benefits

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- This paper:
  - Study standard DMP model with active and passive search
  - Identify restriction implied by perfect substitutability (and reject)
  - Estimate elasticity of substitution < 1, explore implications

What I do, 1/2

(constant marginal efficiency of active search?)

- Formulate standard DMP model w/ active & passive search
  - Active searcher expends effort to find job, passive does not
  - Returns to active and passive search given by fixed parameters
- Derive restriction: active-passive ratio of job-finding probabilities has unit elasticity in average search effort
- Time-series data: elasticity is negative & statistically significant
- Rejection of perfect substitutability in DMP

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- Time-series data: elasticity is negative & statistically significant
- Rejection of perfect substitutability in DMP
- Show from individual-level data: when aggregate active search is high,
  - Active search effort is less effective
  - Penalty from purely passive search is lower

  Suggestive of crowding-out via diminishing returns

What I do, 2/2 (diminishing marginal efficiency of active search.)

- Back to theory: allow for crowding-out via CES aggregator
  - Relax assumption that elas. of subst. btwn active/passive $= \infty$
  - Marginal efficiency of active search no longer constant
- Establish that CES search aggregator is equivalent to linear aggregator with separate matching functions for active and passive search
- Estimate "new" equation for active-passive ratio from the data
  - Recover parameters of aggregator w/ finite elasticity
  - Elasticity of substitution less than one
- Application 1: Optimal policy under Bailey-Chetty formula
- Application 2: Failure of Hosios condition

Why care? Unemployment Insurance during a recession

- Active search of the non-employed is higher during recessions
  - Along both extensive and intensive margins
  - See also Shimer (2004), Mukoyama et al. (2018), etc.
- Elast'y of subst. $< \infty \Rightarrow$ marginal efficiency of active search falls
- UI is less distortionary when marginal efficiency of active search is low
  - Microelasticity falls
  - Independent of surplus splitting mechanism (& wage elasticity)

  Bailey-Chetty formula says you can support a higher level of UI
- Rationalizes estimates indicating only moderate responses of unemployment from recessionary expansions of UI

Theory
(to generate restriction)

Setting

- All jobs generate y_t units of output
- Large measure of firms post v_t vacancies
- Representative family à la Andolfatto (1995) and Merz (1996)
  - Unit measure of workers indexed by i within each family
  - u_t workers are non-employed and search $1 - u_t$ are employed

  Allows for curvature in marginal utility of consumption
- Search of non-employed can be passive and/or active
- Contacts generated through matching function m_t
  - Note: matching efficiency can vary with t

Active and passive search

- Non-employed inelastically provide one unit of passive search
- Non-employed workers choose $s^A_{i,t}$ units of active search, subject to
  - Fixed costs, $\varsigma_{i,t} \sim \Gamma$ drawn *iid* at rate λ
  - Convex costs, $c\left(s^A_{i,t}\right)$
- Flexible to different notions of active search:
  - Intensive & extensive margin: $s^A_{i,t} \in \mathbb{R}_+$ (FMST 2022)
  - Extensive margin only: $s^A_{i,t} \in \{0, 1\}$ (KMRS 2017)

Matching function and job-finding probabilities

- Job-finding rate, $f_{i,t}$

$$f_{i,t} = s_{i,t} \cdot \left(\frac{m_t(s_t, v_t)}{s_t} \right) \qquad (*)$$

with CRS matching function, $m_t(s_t, v_t)$

- Search efficiency, $s_{i,t}$

$$s_{i,t} = \alpha_1 \cdot s^A_{i,t} + \alpha_0 \qquad (**)$$

- Aggregate search efficiency, s_t

$$s_t = \int_i s_{i,t} d\Gamma^u_t$$

Optimal active search

- Flow surplus of employment is increasing in fixed cost of search $\varsigma_{i,t}$
- Can show
  - Active search $s^A_{i,t}$ increasing in fixed cost $\varsigma_{i,t}$ up to some $\check{\varsigma}_t > 0$
  - Workers with $\varsigma_{i,t} > \check{\varsigma}_t$ set $s^A_{i,t} = 0$
- Generates endogenous distributions Γ^u_t and Γ^e_t of workers over $\varsigma_{i,t}$
- Thus, $\Gamma^u_t(\check{\varsigma}_t)$ of non-employed are engaged in active search

▸ Worker's problem ▸ Solution

Restriction: active-passive ratio and average active search

- Restriction in active-passive ratio: $\bar{f}_t^A / \bar{f}_t^P$ and $\bar{s}_A^*$

$$\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1 = \frac{\left(\alpha_1 \cdot \bar{s}_t^{A,*} + \alpha_0\right)\left(\frac{m_t(s_t, v_t)}{s_t}\right)}{\alpha_0 \left(\frac{m_t(s_t, v_t)}{s_t}\right)} - 1 = \left(\frac{\alpha_1}{\alpha_0}\right) \cdot \bar{s}_t^{A,*}$$

  from eqn's $(*)$ and $(**)$

- Unit elasticity in $\bar{s}_t^{A,*}$
- All other aggregate quantities drop out!
  - Note: match efficiency differenced out
- Similar restriction appears in Krusell, Mukoyama, Rogerson, and Sahin (2017, AER) & Faberman, Mueller, Sahin, and Topa (2022, ECTA) & …

Bringing the restriction to the data

CPS, 1996-2019

- Starting in 1996, CPS records following for jobless respondents:
  - Whether the respondent would be willing to accept a job
  - Whether the worker is engaged in nine methods of active search
  - If # search methods $= 0$, why no active search?
- Non-employed worker willing to accept a job is
  - Active searcher if # search methods > 0
  - Passive searcher: if # search methods $= 0$ & “able” to accept work
- # of search methods highly correlated with time spent searching (Mukoyama, Patterson, and Sahin 2018) $\Rightarrow$ measure of search effort
- Note: excluding temporary-layoff for practical and conceptual reasons

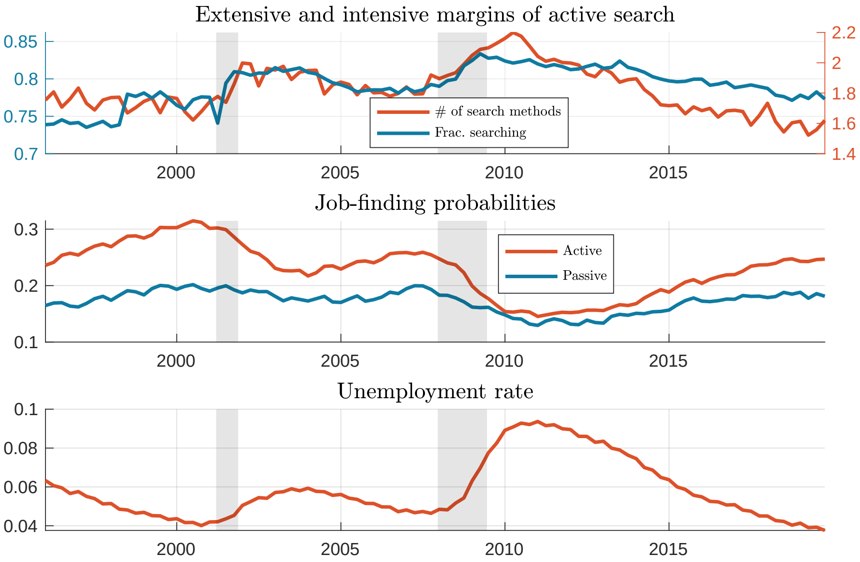
Search and job-finding probabilities

The active-passive ratio of job-finding prob's and aggregate search

	Frac. searching	# search methods	A-P ratio in JFP's
mean(x)	0.8	1.9	$1/0.75$
std(x)/std(Y)	1.7	2.8	9.2
corr(x, Y)	-0.69	-0.60	0.50

- Both frac. searching & # of search methods is countercyclical
  - See also Shimer (2004), Faberman and Kudlyak (2016), Mukoyama, Patterson, and Sahin (2018)
- Active-passive ratio is procyclical

Search and job-finding probabilities



Testing the restriction

- Recall restriction:

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\alpha_1}{\alpha_0}\right) + \log \bar{s}_t^{A,*}$$

  Theory predicts unit elasticity

- Estimated elasticity from data: -5.47 (SE$=0.765$)
- Robust to:
  - Restricting active searchers to low duration of unemployment
  - Disaggregating by gender, age, education, region, marital status …
- Qualitative rejection of DMP w/ perfect substitution of active & passive

An unrestricted
CES search aggregator

What went wrong

- Reject restriction from perfect substitution of active/passive
  - Perfect substitutes $\iff$ CES with elasticity of subst. $= \infty$
- Additional findings from micro-level data:
  - Active search less effective when aggregate search is higher
  - Penalty to passive searchers decreasing in aggregate search
- Suggests efficiency of active search diminishing in aggr. active search
  - w/ CES, requires elasticity of subst. $< \infty$
- Next: estimate parameters of CES over active/passive ratio with unrestricted elasticity

CES aggregator for search effort

- Aggregate search effort s_t given by CES aggregator over $s_{A,t}$ and $s_{P,t}$

$$s_t = \left(\omega \, s_{A,t}{}^{\rho} + (1-\omega)s_{P,t}^{\rho}\right)^{\frac{1}{\rho}}$$

- Aggregate active & passive search satisfy

$$s_{A,t} = \int^{\check{\varsigma}_t} s_{i,t}^{A} d\Gamma_t^u = (\Gamma_t^u(\check{\varsigma}_t)u_t) \cdot \bar{s}_{A,t}^{*}, \quad s_{P,t} = \int d\Gamma_t^u = u_t$$

- $ME_{A,t}$ and $ME_{P,t}$ are marginal efficiencies of active and passive search

$$ME_{A,t} = \frac{\partial s_t}{\partial s_{A,t}} = \omega \cdot \left(\frac{s_t}{s_{A,t}}\right)^{1-\rho}, \quad ME_{P,t} = \frac{\partial s_t}{\partial s_{P,t}} = (1-\omega) \cdot \left(\frac{s_t}{s_{P,t}}\right)^{1-\rho}$$

What is a CES search aggregator?

- Equivalence: separate submarkets for active and passive search

$$m_t(s_t, v_t) = m_t\left(ME_{A,t} \cdot s_{A,t}, \alpha_t \cdot v_t\right) + m_t\left(ME_{P,t} \cdot s_{P,t}, (1-\alpha_t) \cdot v_t\right)$$

with

$$\alpha_t = \alpha(s_{A,t}/s_{P,t}) = \frac{ME_{A,t} \cdot s_{A,t}}{s_t} = \frac{s_{A,t}^{\rho}}{s_{A,t}^{\rho} + s_{P,t}^{\rho}}, \qquad \rho \leq 1$$

- Result obtains through constant returns
- Vacancy share of active search α_t analogous to factor share

Returns to search

- The job-finding probability $f_{i,t}$ of a worker with search efficiency $s_{i,t}$ is

$$f_{i,t} = s_{i,t} \cdot \left(\frac{m_t\left(s_t, v_t\right)}{s_t} \right)$$

- The search efficiency $s_{i,t}$ of a worker supplying $s^A_{i,t}$

$$s_{i,t} = ME_{A,t} \cdot s^A_{i,t} + ME_{P,t} \cdot 1$$

  by linear homogeneity of the CES search aggregator

- Nests prior case when $\rho = 1$:

$$s_{i,t} = \Big(\underbrace{\omega}_{\equiv \alpha_1} s^A_{i,t} + \underbrace{(1-\omega)}_{\equiv \alpha_0} \Big).$$

Restriction from theory, redux

- Relative job-finding probabilities, active vs. passive search

$$\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1 = \frac{\left(ME_{A,t} \cdot \bar{s}_t^{A,*} + ME_{P,t}\right)\left(\frac{m_t(s_t,v_t)}{s_t}\right)}{ME_{P,t}\left(\frac{m_t(s_t,v_t)}{s}\right)} - 1$$

$$= \left(\frac{\omega}{1-\omega}\right)\left(\frac{1}{\Gamma_t^u(\check{\varsigma}_t)\bar{s}_t^{A,*}}\right)^{1-\rho} \cdot \bar{s}_t^{A,*}$$

- Thus,

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1-\omega}\right) + (\rho - 1) \cdot \log \Gamma_t^u(\check{\varsigma}_t) + \rho \cdot \log \bar{s}_t^{A,*}$$

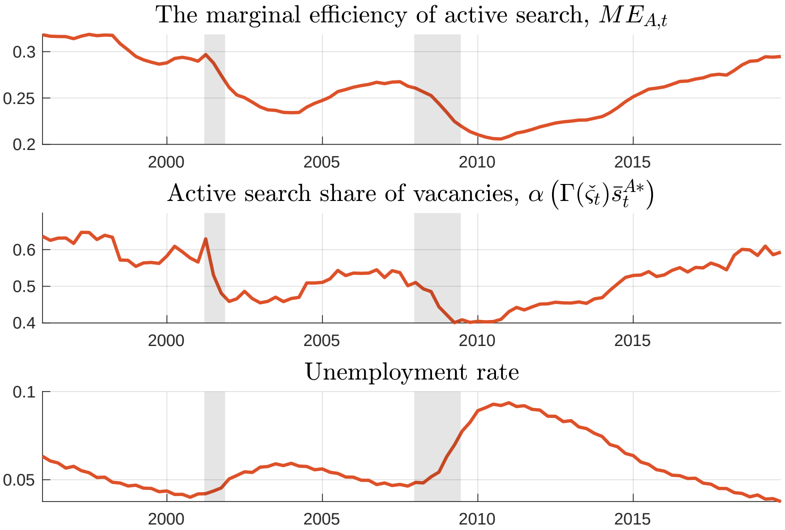
- Return to data, estimate ω and ρ, test restriction in ρ

Regression estimates

	(1)	(2)	(3)
Fraction searching	-7.31	-5.281	-3.819
	(1.426)	(1.606)	(0.5549)
# of search methods	–	-1.927	-2.812
		(0.8609)	(0.5549)
Constant	-0.684	0.489	1.140
	(0.4257)	(0.6392)	(0.1547)
Additional controls		Time trend	
Constrain $\beta_{\text{Frac}} - 1 = \beta_{\#}$?	N/A	No	Yes
F-test	$p(\rho = 1)$	$p(\beta_{\text{Frac}} + 1 = \beta_{\#})$	$p(\rho = 1)$
	=0.0000	=0.2799	=0.0000
N	261	261	261
Implied ρ	-8.308	–	-2.819
Implied ω	0.335		0.758
Elasticity of substitution	0.107		0.268

CPS, 1996-20019

Backing out the marginal efficiency of active search



Takeaway

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1-\omega}\right) + (\rho - 1)\cdot \log \Gamma_t^u(\check{\varsigma}_t) + \rho \cdot \log \bar{s}_t^{A,*}$$

- Reject restriction $\rho = 1$ (i.e., existing framework)
- Fail to reject restriction $\beta_{\Gamma(\check{\varsigma})} + 1 = \beta_{\bar{s}^{A,*}}$ (i.e., unrestricted framework)
- Elasticity of substitution $\frac{1}{1-\rho}$ falls in range $(\frac{1}{10}, \frac{1}{4})$
  - Indicates that active and passive search are “complements”
  - Thus active search vacancy share declining in active search

Application 1:
Bailey-Chetty Formula

Appl. 1) Bailey-Chetty formula

- Consider simple Bailey-Chetty formula,

$$\epsilon_R = \left(\frac{U'(c^u)}{U'(c^e)} - 1 \right) \tag{BC}$$

  where ϵ_R is the micro-elasticity of unempl. w/r.t. replacement rate R

- Existing result: if wages are perfectly rigid ($+$ other conditions), macro-elasticity equals micro, and constant R is optimal
  - e.g., Landais et al. (2018)
  - Constant R depends on assumption of constant micro-elasticity ϵ_R

- But ϵ_R is proportional to marginal efficiency of active search

- Thus, microelasticity decreases during recessions $\Rightarrow R \uparrow$

Appl. 1) Bailey-Chetty Formula, cont.

- Assume $U(c) = \log c$
- Define the *consumption decline upon job-loss*: $\Delta_t = \frac{c_t^e - c_t^u}{c_t^e}$
- Optimal Δ_t lower than average during recessions via microelasticity

Application 2:
Unexplored externality

Appl. 2) Unexplored externality

$$rU_i = \max_{s_i^A} \left\{ \frac{b - \varsigma_i \cdot \mathbb{I}\left\{s_i^A > 0\right\} - c\left(s_i^A\right)}{\mu} \right.$$
$$\left. + \left(ME_A \cdot s_i^A + ME_P\right) \cdot \left(\frac{m\left(s, \upsilon\right)}{s}\right) \cdot \left(V_i - U_i\right) - \dot{U}_i \right\}$$

- Congestion externality: searchers fail to internalize how $s_{A,i}$ affects s
- Here: searchers also fail to internalize how $s_{A,i}$ affects ME_A and ME_P
- $s^*_{A,i} \uparrow \Rightarrow ME_A \downarrow$ and $ME_P \uparrow$

Appl. 2) Unexplored externality, con't

- Optimal search, worker's problem:

$$s^*_{A,i} = (c')^{-1}\left(ME_A \cdot f(\theta) \cdot \psi_i\right)$$

  where ψ_i is the marginal value to the HH of having agent i employed

- Optimal search, Planner's problem:

$$s^{SP}_{A,i} = (c')^{-1}\left(ME^{SP}_A \cdot f(\theta^{SP}) \cdot \psi^{SP}_i + \underbrace{\frac{\partial ME^{SP}_A}{\partial s_A} \cdot cov(s^{SP}_{A,i}, \psi^{SP}_i)}_{<0}\right)$$

  where ψ^{SP}_i is the marginal social value of having agent i employed

- Two allocations only coincide if
  1. No persistent heterogneity in fixed cost of search ($\lambda \to \infty$)
  2. Constant marginal efficiency of active search, ME_A

Concluding remarks

Conclusion

- Finite elasticity of substitution between active and passive search
- Thus, dynamics of unemployment and job-finding rates depend on aggregate composition of active/passive search
- Reinforces message from Elsby, Hobijn, and Sahin (2015), Krusell et al. (2017), Faberman et al. (2022), and more:

  We need to incorporate non-participation & passive search into more of our models to better understand unemployment

Extra slides

Problem of the unemployed

- Annuity value of unemployment:

$$rU_{i,t} = \max_{s_{i,t}^A} \left\{ \frac{b_t - \varsigma_i \cdot \mathbb{I}\left\{ s_{i,t}^A > 0 \right\} - c\left(s_{i,t}^A \right)}{\mu_t} + \left(\alpha_0 + \alpha_1 \cdot s_{i,t}^A \right) \cdot \left(\frac{m_t\left(s_t, v_t \right)}{s_t} \right) \cdot \left(V_{i,t} - U_{i,t} \right) - \dot{U}_{i,t} \right\}$$

- Marginal utility of consumption, μ_t
- Flow value of leisure, b_t
- Values of employment and unemployment, $V_{i,t}$ and $U_{i,t}$
- $\dot{U}_t \neq 0$ given jump process for $\varsigma_{i,t}$, etc

Optimal active search

- Optimal quantity of active search (intensive margin):

$$s_{i,t}^{A,*} = (c')^{-1}\left(\mu_t \cdot \alpha_1 \cdot \left(\frac{m_t(s_t, v_t)}{s_t}\right)(V_{i,t} - U_{i,t})\right) \quad \text{when } \varsigma_{i,t} < \check{\varsigma}_t$$

- Optimal participation in active search (extensive margin):

$$\varsigma_{i,t} \leq -c(s_{i,t}^{A,*}) + \alpha_1 \cdot s_{i,t}^{A,*} \cdot \left(\frac{m_t(s_t, \upsilon_t)}{s_t}\right) \cdot \mu_t \cdot (V_{i,t} - U_{i,t}) \qquad (\dagger)$$

where $\check{\varsigma}_t$ defined by $\varsigma_{i,t}$ s.t. (†) holds with equality

When is active search most effective?

	Indicator variable for moving to employment in subsequent period			
	(1)	(2)	(3)	(4)
# of search methods	−0.002	0.113	0.057	–
	(0.0004)	(0.0058)	(0.0079)	
# of search methods × aggr. active search	–	−0.060	−0.031	–
		(0.0030)	(0.0041)	
$\mathbb{I}\{\#\text{ search methods} = 0\}$	−0.040	−0.036	−0.261	−0.414
	(0.0013)	(0.0013)	(0.0192)	(0.0215)
$\mathbb{I}\{\#\text{ search methods} = 0\}$ × aggr. active search	–	–	0.120	0.479
			(0.0101)	(0.0270)
N	865079	865079	865079	865079
Time fixed effects?	Yes	Yes	Yes	Yes
Region fixed effects?	Yes	Yes	Yes	Yes

Sample of active and passive searchers, 1996-2019

Incl. controls for education, quartic for age, gender, race, and marital status

- Search is less effective when aggregate search is higher
- Penalty to to purely passive search lower when aggregate search is higher