THE MARGINAL EFFICIENCY OF ACTIVE SEARCH

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Abstract. During recessions, a greater fraction of non-employed workers who want work actively search for a job. Simultaneously, the job-finding probabilities of such non-employed active searchers converge to those of the non-employed who want a job but are not engaged in any form of active search. I document these findings and show that they are symptomatic of a form of "crowding-out" of active search that has thus far gone unrecognized in the literature. I estimate a declining marginal efficiency of active search, and I establish that active search plays a less important role for finding a job during a recession.

A non-employed worker willing to accept employment can find it by actively seeking out a job, or by passively waiting for a job to find them. The *active non-employed* find a job at a rate that is on average one-third higher that of the *passive non-employed*. But the ratio of job-finding probabilities of the active and passive non-employed is cyclical. During a recession, the job-finding probabilities of the active non-employed decline relative to the passive non-employed—despite the fact that a greater fraction of the non-employed are engaged in active search in a recession, and the intensity of active search among the active non-employed is higher. Thus, the premium in the job-finding probability associated with active search declines with the aggregate quantity of active search. These findings cannot be explained by other factors, such as cyclicality in the composition of workers in active and passive non-employment.

This paper is the first to document these properties of the active-passive ratio in job-finding probabilities. I show that the findings are consistent with a "crowding-out" of active search, whereby active search acts as a strategic substitute: given a vacancy posting with a pool of known candidates (i.e., passive searchers), the probability that the vacancy turns into a job for an outside applicant is declining in the total number of outside applicants. Thus, the job-finding probability from active search relative to purely-passive search is decreasing in the total quantity of active search. While such a crowding-out of active search may seem intuitive, it is ruled out in the existing

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literature under the ubiquitious assumption that active and passive search enter the matching function as perfect substitutes.[1]

As I show, a simple relaxation of this assumption allows a canonical three-state model of unemployment, employment, and labor force inactivity à la Krusell et al. (2017) to be consistent with the paper's empirical findings through a diminishing *marginal efficiency of active search*, so that an increase in the fraction of active-nonemployed among non-employed who want work reduces the rate at which an increase in active search effort increases search efficiency (and thus also job-finding rates).

Relaxing the three-state model to accomodate the crowding-out of active search shown in the data generates additional implications from this class of models for both the dynamics of unemployment and the role of policy. A recessionary increase in active search reduces the marginal efficiency of active search, implying that active search contributes less to the probability of finding a job. Thus, the convergence in job-finding probabilities of the active and passive non-employed reflects a reduced role in active search for finding a job.

A diminishing marginal efficiency of active search carries important policy implications. Under a standard Bailey-Chetty formula, the marginal efficiency of active search is taken to be constant, and thus the optimal level of benefits is invariant. Here, however, the marginal efficiency of active search declines during recessions: thus, although UI may still have a disincentive effect on active search, any given reduction in active search does less to reduce job-finding rates, and therefore is less distortionary. Therefore, a recessionary expansion of UI can be taken as optimal.

I begin my analysis by developing a theoretical restriction that must hold in the absence of a "crowding-out" of active search. I study the labor-supply block of a three-state model of inactive non-employment, unemployment, and employment à la Krusell et al. (2017), where active and passive search are taken as perfect substitutes. Workers incur random but persistent fixed costs of search, and the total cost of search is convexly increasing in its intensity. Although the model incorporates multiple dimensions of heterogeneity, I establish the endogenous active search decision of a worker as a sufficient statistic for the worker's job-finding probability, taking the aggregate state as given.

Under this general framework, I develop a restriction that must hold in the absence of crowding-out: the active-passive ratio in job-finding probabilities (minus one) has a unit elasticity in the active search effort of the non-employed. No other aggregate variables enter the restriction. Although the restriction takes a simple form, the restriction is implied across a broad class of three-state models where active and passive search are taken as perfect substitutes, e.g. Krusell et al. (2017) and Faberman et al. (2022).[2]

[1]To my knowledge, this assumption is first introduced in Blanchard and Diamond (1990, pg. 34).

[2]I highlight the role of the assumption in Krusell et al. (2017) and Faberman et al. (2022) due to their influence in the literature. However, the assumption is ubiquitious to papers on three-state models of employment, unemployment, and non-participation.

Then, taking the theoretical restriction to the post-1994 CPS data, I show that the restriction can be easily rejected. Focusing the analysis on the active non-employed

I take the theoretical restriction to the post-1994 CPS data, where a survey redesign allows for clear identification of the passive non-employed and offers a measure of active search effort. There, I focus my analysis on two seemingly similar populations of non-employed workers: the active non-employed (non-employed workers who are willing to accept a job and are engaged in active search) and the passive non-employed (non-employed workers who are willing to accept a job but are not engaged in active search). To the extent that such workers find jobs in a manner that is to some degree similar and thus can be viewed as substitutable, one should expect it difficult to reject the restriction implied under the absence of crowding-out.[3]

Rather than recovering a unit elasticity of the active-passive ratio in job-finding probabilities with respect to the average active search effort of the active non-employed—as implied under the no-crowding-out restriction—I estimate an elasticity that is negative, large in magnitude, and precisely estimated, at nearly negative six. Thus, during recessions, when a greater fraction of the non-employed who want work are engaged in active search, the average active search of the active non-employed increases; but the job-finding probability associated with active search decreases relative to that of the passive non-employed. Thus, the data shows that the premium in job-finding probabilities from active search is *decreasing* with average active search, instead of *increasing* (as predicted by the theory).

A possible concern is that the violation of the restriction implied under no-crowding-out comes from some artifact of the data; for example, resulting from cyclical heterogeneity in the composition of the active and passive non-employed. I offer evidence that this is not so. The estimate of a large and negative elasticity of the active-passive ratio in job-finding probabilities to active search effort is robust to the inclusion of trend and cyclical variation in the active-passive ratio; the allowance of time-varying marginal efficiencies of active and passive search; and shift-share estimates to control for the cyclical heterogeneity in the composition of active and inactive non-employed. Thus, the data permit a qualitative rejection for the predictions of a standard three-state DMP model where active and passive search are taken as perfect substitutes.

Thus, I return to the model, but relax a single assumption to allow for a "crowding-out" of active search. Rather than assume that active and passive search enter the matching function as perfect substitutes, I allow the two search inputs to enter the matching function through an unrestricted CES aggregator. I derive an equation for the active-passive ratio of job-finding probabilities under the unrestricted CES aggregator, which is shown to include an additional term for the fraction of non-employed engaged in active search: In logarithms, average active search effort and the additional term enter the equation for the active-passive ratio with coefficients that are linearly decreasing in the elasticity of substitution. The unrestricted equation allows for an active-passive ratio in job-finding probabilities that is decreasing in the

[3]For a view embracing the notion that these two groups of non-employed are fundamentally the same, see Hall (2006), where both are included in a single expanded measure of unemployment.

quantity of active search effort via an elasticity of substitution between active and passive search that is less than one. Moreover, the dependence of the two coefficients on the elasticity of substitution offers an over-identifying restriction under which the model can be rejected.

Returning to the data, I am unable to reject the over-identifying restrictions implied under the unrestricted CES aggregator over active and passive search. I estimate the structural parameters of the CES aggregator that are necessary for computing the elasticity of substitution, as well as the function that gives the time-varying marginal efficiencies of active and passive search. I recover an elasticity of substitution of around one-fourth. Thus, rather than being perfect substitutes with an infinite elasticity of substitution, as assumed in the existing literature, active and passive search are estimated to be close complements. For the non-employed worker, this means that active search is a strategic substitute: the job-finding rate from active search relative to inactive search is greater when fewer other workers are engaged in active search. To the firm, this can be understood as the consequence of a hiring process that favors a relatively stable ratio of new hires out of referrals relative to outside applications.

In the remainder of the paper, I consider the positive and normative implications of my empirical findings. To sharpen the analysis, I begin by proving a simple representation theorem: a CES aggregate over active and passive search entering a *single* matching function is equivalent to a setting where active and passive search are intermediated by *separate* matching functions, with each search input weighted by its respective marginal efficiency, and market tightness equated across matching functions. I derive formulas necessary for calculating the "active search share of vacancies," the notional share of vacancies separately allocated to the active search submarket.

Together with the estimated parameters of the CES aggregator, I use these quantities to explain how a recessionary increase in active search can lead to a decline in the active-passive ratio in job-finding probabilities. In particular, I show that the both the marginal efficiency of active search and the active search share of vacancies fall dramatically during a recession. For example, during the Great Recession, the active search share of vacancies and the marginal efficiency of active search each decline by more than 50 percent, similar in magnitude to the drop in aggregate match efficiency estimated by Gavazza, Mongey and Violante (2018). Thus, even though recessions are periods when the fraction of the non-employed engaged in active search is at its zenith — and when the active search effort of such workers is at its peak — recessions are also periods where active search plays the least important direct role for matching a worker with a job.

Then, I turn to the normative implications of my estimates. A common concern with unemployment insurance is that it disincentivizes active search: At the extreme, the provision of UI may incentive workers who would otherwise be among the active non-employed to quit active search and instead find jobs at the lower rate associated with the passive non-employed. The potential disincentive cost of UI can be related to the active-passive ratio in job-finding probabilities: a larger active-passive ratio implies a larger increase in unemployment if the average active searcher from non-employment stops searching actively. The paper's finding that the active-passive ratio in job-finding probabilities falls during a recession thus might suggest that the disincentive cost of UI is lower during a recession.

To offer a structural foundations to this intuition, I study the implications of my estimates for the Bailey-Chetty formula for optimal UI. The Bailey-Chetty formula describes the optimal replacement rate chosen by a policy maker wishes to smooth consumption across the unemployed and employed, but who is constrained by budget reductions associated with the disincentive effect of expanding UI on active search. The micro-elasticity of unemployment with respect to the replacement rate, describing both the search response to a change in UI and the further implied response for the probability of finding a job, enters the formula as a crucial statistic. But whereas the micro-elasticity is taken to be fixed in the existing literature, it shown here to be proportional to the elasticity of total search efficiency with respect to active search, which may vary over time. Combining data with the recovered parameters of the CES aggregator over active and passive search, I estimate the elasticity to be decreasing with the aggregate quantity of active search, and thus decreasing during a recession. Given the decreased responsiveness of unemployment rate to variations in active search effort, a policy maker can achieve the constrained efficient allocation with a lower replacement rate during recessions. Intuitively, even if the disincentive effect of UI goes unchanged during recessions, the diminished marginal efficiency of active search implies that the disincentive effect of UI matters less for the probability that a worker finds a job.

Finally, I consider the normative implications of my estimates of a declining marginal efficiency of active search under a three-state model, studying the problem of a social planner who wishes to maximize aggregate consumption. I show that a diminishing marginal efficiency of active search gives rise to a new externality associated with the crowding-out of active search. A social planner will induce workers to internalize the effect of their search effort on the aggregate marginal efficiency of active search. The importance of this crowding-out externality depends on the degree of concavity of aggregate search efficiency over active and passive search effort; and the amount of heterogeneity in the marginal social value offered by different types of workers. Intuitively, when some workers offer a much higher marginal social value of employment than other workers, the social planner prescribes less search, as not to overly reduce the marginal efficiency of active search and impede workers with a high marginal social value of employment from finding a job.

Related Literature. This paper touches on several distinct areas of the literature.

The paper provides additional evidence for the importance of non-participation in understanding the dynamics of unemployment. Although models of unemployment in the tradition of Diamond, Mortensen, and Pissarides typically exclude a role for non-participation (or the coexistence of active and passive search), the importance of non-participation for broader labor market dynamics is well-documented. For example, Blanchard and Diamond (1989) estimate that half of all new hires from non-employment originate from inactivity. In subsequent work, Blanchard and Diamond (1990) document the empirical relevance of the inactive non-employed for explaining the cyclical behavior of vacancies and new hires. In more recent research, Faberman et al. (2022) document the importance of unsolicited employer contacts for job-finding, empirically validating that workers do indeed find jobs via "passive" search.

Accordingly, a recent literature has emerged to incorporate a role for labor force inactivity (including passive search) into frictional models of equilibrium unemployment, including Krusell, Mukoyama, Rogerson and Şahin (2017, 2020), Cairó, Fujita and Morales-Jiménez (2022), Faberman, Mueller, Şahin and Topa (2022), and Ferraro and Fiori (2022). The paper here contributes this literature in documenting the joint cyclical dynamics of job-finding among the active and passive non-employed. Furthermore, the paper demonstrates that, by relaxing the assumption of perfect substitution of active and passive search, three-state models à la Krusell et al. (2017) and Faberman et al. (2022) can accommodate the central empirical findings of the paper. In doing so, the paper identifies new channels to which these models are crucial for understanding labor market dynamics and policy.

This paper also relates to the important empirical literature evaluating the response of unemployment to variation in unemployment insurance benefits over the business cycle. Chodorow-Reich et al. (2018) isolate UI extensions triggered by measurement error in recorded unemployment to estimate the causal impact of an increase in UI durations during the Great Recession. Even though benefit increases were substantial, the estimates from Chodorow-Reich et al. (2018) imply that they generated an almost negligible increase in unemployment. Similar estimates from Rothstein (2011), Farber and Valletta (2015), and Kroft and Notowidigdo (2016) imply that increases in UI benefits during recessions have smaller effects on job-finding probabilities than during normal times. Such estimates smaller distortions from expansions of UI during a recession can be understood through the lower marginal efficiency of active search: even if the disincentive effect of UI on search goes unchanged during a recession, active search is less important for job-finding during a recession due to the reduced marginal efficiency of active search.

The paper similarly relates to the literature studying optimal unemployment insurance over the business cycle from a model of equilibrium unemployment, including Mitman and Rabinovich (2015) and Landais et al. (2018a,b). These papers typically take the micro-elasticity summarizing the labor supply response to UI and the implied response for unemployment as fixed. Instead, they consider the cyclical behavior of optimal UI using a macro-elasticity that summarizes how job-creation responds to UI depending on aggregate conditions. This paper largely abstracts from the macro-elasticity, but documents procyclical variation in the micro-elasticity and its implications for optimal UI. Notably, the channel identified by this paper operates independently from market tightness, and thus the mechanism explored by the paper is independent of particular assumptions made about surplus splitting and the cyclicality of wages, two issues that divide the literature on optimal UI.[4] The paper also adds to the existing literature in its focus on the participation margin and the implications of counter-cyclical active search effort for unemployment insurance.

This paper explains the decline in the active-passive ratio in job-finding probabilities through the increase in active search during recessions. Direct evidence of countercyclical active search along the extensive and intensive margins comes from Osberg (1993), Shimer (2004), Elsby et al. (2015), Mukoyama et al. (2018), and Faberman and Kudlyak (2019). Accordingly, Krusell et al. (2017) and Cairó et al. (2022) study models with a countercyclical extensive margin of active search to explain countercyclical flows from non-participation to unemployment. Yet, perhaps because the textbook model endogenous search à la Pissarides (2000) generates procyclical search, some researchers have been reluctant to incorporate countercyclical active search from non-employment into modeling frameworks.

Thus, the model of this paper incorporates an explicit channel by which active search among the non-employed may decrease or increase during a recession: On the one hand, recessionary decline in job-finding probabilities lowers the return to active search, and so job-seekers may search less (or not at all). On the other hand, if the marginal utility of consumption rises during recessions, workers place less value on the additional leisure afforded through unemployment, and therefore may search more. As the paper documents, the strength of this latter channel is determined by the opportunity cost of employment à la Chodorow-Reich and Karabarbounis. Chodorow-Reich and Karabarbounis (2016) carefully measure the opportunity cost of employment using micro and aggregate data on government transfers, taxes, take-up of benefits, consumption, and hours of work. They find the opportunity cost to be highly procyclical. Hence, from the model developed here, the findings of Chodorow-Reich and Karabarbounis (2016) constitute additional (albeit indirect) supporting evidence for countercyclical search of the non-employed.

The paper documents a crowding-out of active search from direct empirical evidence of a finite elasticity of substitution between active and passive search. Several recent papers propose similar "crowding-out" phenomenon affecting aggregate unemployment, albeit by different channels: For example, Michaillat (2012) studies a model of rigid wages and capacity constraints where job-rationing drives the recessionary increase in unemployment; Engbom (2021) studies a model in which the matching process exhibits diminishing returns to scale in the number of job applicants; and Mercan et al. (2022) propose a theory of diminishing returns to scale of output in the number of new hires. The focus of these papers is largely to account for the volatility of unemployment and market tightness. In contrast, the framework here takes the dynamic behavior of market tightness as given, instead seeking to account for the joint cyclical behavior of job-finding for workers with different levels of labor market attachment.

Finally, the paper relates to the literature using micro data to gain additional insights into the process by which workers match with jobs, e.g. Davis et al. (2013),

[4]See Section IV of Landais et al. (2018b) for an helpful discussion of the importance of these issues.

Gavazza et al. (2018), and Mongey and Violante (2019). In this regard, the paper especially draws on empirical evidence from Faberman et al. (2022) and Lester et al. (2021) on the process by which workers receive job offers without actively searching.

1. The active-passive ratio in job-finding probabilities: theory

Here, I study a standard DMP model that incorporates endogenous extensive and intensive margins of active search: non-employed workers face a choice of whether and how much to search. Crucially, active and passive search of the non-employed enter the matching function as perfect substitutes, and thus there is no crowding-out of active search. Thus, I am able to use the model to derive a theoretical restriction that holds in the absence of crowding out. The restriction relates the active-passive ratio in job-finding probabilities to the average active search effort of the active non-employed, where the active-passive ratio in job-finding probabilities is defined as the average job-finding probability of a non-employed worker engaged in active search divided by the average job-finding probability of a non-employed worker not engaged in active search. As will become apparent, the desired restriction is sufficiently general that it can be derived without placing restrictions on vacancy posting or surplus splitting. Hence, I consider the model in partial equilibrium, leaving the surplus-splitting mechanism and entry conditions unspecified.

1.1. **Environment.** Time is discrete, and there is an infinite horizon. There is a unit measure of households and a large measure of firms. Each household is comprised of a continuum of workers who perfectly insure each other against labor market risk. Workers derive utility from consumption and leisure, have time separable preferences, discount the future by a constant factor β, and sacrifice leisure to work or engage in active search. Workers are heterogeneous both their productive ability and their cost of active search.

A measure Υ_t of workers are matched with firms. A matched worker and job produce $z_t \cdot x$ units of the consumption good, where z_t is aggregate productivity at time t, and x is the fixed productivity of the worker, distributed by the invariant CDF Γ_x.

A measure $1 - \Upsilon_t$ of non-employed workers enjoy flow leisure ψ and inelastically provide a single unit of passive search. A non-employed worker i with characteristics (x, ς) at time t chooses a quantity $s_{A,i,t}$ units of active search, subject to fixed and variable costs of search denominated in foregone leisure, where ς gives the worker's fixed cost of active search. With probability λ at the beginning of every period, a worker draws a new fixed cost ς' from an invariant distribution Γ_ς. The second component of the cost of active search is strictly convex in the total quantity of active search $s_{A,i,t}$ provided by the worker.

The worker's flow utility can be expressed as a function of the worker's consumption, employment status, and active search decision. In particular, given a fixed cost of search ς, the flow utility of a worker is given by

$$\mathcal{U}(c, s_A, e) = \frac{c^{1-\sigma}-1}{1-\sigma} + \left(\psi - \varsigma \cdot \mathbb{I}\{s_A > 0\} - \chi \cdot \frac{s_A^{1+\varkappa}}{1+\varkappa}\right) \cdot \mathbb{I}\{e = 0\} \tag{1}$$

where σ is the coefficient of relative risk aversion, ψ is the flow value of leisure, χ is a scaling term for the disutility of active search, and the parameter $\varkappa > 0$ describes the convexity of active search costs.

1.2. **The search environment.** Firms post vacancies to match with workers, and workers engage in active and passive search to find vacancies and match with firms.

The number of new matches within a given period is determined by an aggregate matching function with constant returns to scale over aggregate search efficiency $\boldsymbol{s}_t$ and vacancies υ_t, $m_t(\boldsymbol{s}_t, \upsilon_t)$.[5] Then, the aggregate job-finding probability per unit of search efficiency is given by the ratio of new matches to units of aggregate search efficiency:

$$f_t = \frac{m_t(\boldsymbol{s}_t, \upsilon_t)}{\boldsymbol{s}_t} \tag{2}$$

Similarly, the probability q_t that a firm fills a vacancy at time t is given by the ratio of new matches to vacancies,

$$q_t = \frac{m_t(\boldsymbol{s}_t, \upsilon_t)}{\upsilon_t} \tag{3}$$

A worker i with characteristics (x, ς) at time t vary in search efficiency $s_{i,t}$ by their endogenously chosen active search $s_{A,i,t}$. Individual search efficiency $s_{i,t}$ is a linear combination of the worker's endogenously chosen active search $s_{A,i,t}$ and the inelastically supplied unit of passive search,

$$s_{i,t} = \omega \cdot s_{A,i,t} + (1-\omega) \cdot 1, \tag{4}$$

where active search receives weight ω and passive search receives weight $1-\omega$. Thus, the job-finding probability $f_{i,t}$ of a worker i with characteristics (x, ς) providing $s_{i,t}$ units of search efficiency at time t is given as

$$\begin{aligned} f_{i,t} &= s_{i,t} \cdot \left(\frac{m_t(\boldsymbol{s}_t, \upsilon_t)}{\boldsymbol{s}_t}\right) \\ &= s_{i,t} \cdot f_t \end{aligned} \tag{5}$$

Note, the multiplicative structure of equation (5) guarantees that worker specific job-finding probabilities, $f_{i,t}$, integrate to the number of total new matches in period t.[6]

Aggregate search efficiency $\boldsymbol{s}_t$ is defined as linear composite over aggregate active search $\boldsymbol{s}_{A,t}$ and aggregate passive search $\boldsymbol{s}_{P,t}$:

$$\boldsymbol{s}_t = \omega \cdot \boldsymbol{s}_{A,t} + (1-\omega) \cdot \boldsymbol{s}_{P,t}. \tag{6}$$

[5]Note, the matching function is indexed by t. Thus, components of the matching function (such as match efficiency) might vary over time.

[6]Equation (5) is defined exactly as in equation (5.2) of Pissarides (2000, page 125).

The linear form of the aggregator implies that active and passive search are perfect substitutes. Given an individual i with characteristics (x, ς) at time t, denote $s_{A,t}(x, \varsigma) \equiv s_{A,i,t}$. Then,

$$\boldsymbol{s}_{A,t} = \int\limits_{X \times \mathcal{C}} s_{A,t}(x, \varsigma) \cdot d\Gamma_t^{ne}(x, \varsigma) \tag{7}$$

$$\boldsymbol{s}_{P,t} = \int\limits_{X \times \mathcal{C}} 1 \cdot d\Gamma_t^{ne}(x, \varsigma) \tag{8}$$

where $\Gamma_t^{ne}(x, \varsigma) \equiv \Gamma_x(x)\Gamma_\varsigma(\varsigma) - \Upsilon_t(x, \varsigma)$ gives the distribution of non-employed workers.

Define the marginal efficiency of active search $ME_{A,t}$ at time t as the derivative of aggregate search $\boldsymbol{s}_t$ with respect to aggregate active search $\boldsymbol{s}_{A,t}$, and define the marginal efficiency of passive search $ME_{P,t}$ similarly. Then,

$$ME_{A,t} \equiv \frac{\partial \boldsymbol{s}_t}{\partial \boldsymbol{s}_{A,t}}, \quad ME_{P,t} \equiv \frac{\partial \boldsymbol{s}_t}{\partial \boldsymbol{s}_{P,t}}. \tag{9}$$

Simple inspection of equation (6) reveals that the marginal efficiency of active and passive search, $ME_{A,t}$ and $ME_{P,t}$, are given by the fixed quantities ω and $1 - \omega$.

Finally, the law of motion for $\Upsilon_t(x, \varsigma)$ can be written as

$$d\Upsilon_{t+1}(x, \varsigma') = \begin{cases} \lambda \cdot d\Gamma_x(x) d\Gamma_\varsigma(\varsigma) \cdot \int d\Upsilon_{t+}(\check{x}, \check{\varsigma}) + (1 - \lambda) \cdot d\Upsilon_{t+}(x, \varsigma) & \text{if } \varsigma' = \varsigma \\ \lambda \cdot d\Gamma_x(x) \Gamma_\varsigma(\varsigma') \cdot \int d\Upsilon_{t+}(\check{x}, \check{\varsigma}) & \text{if } \varsigma' \neq \varsigma \end{cases} \tag{10}$$

with

$$d\Upsilon_{t+}(x, \varsigma) = f_t(x, \varsigma)\Big(d\Gamma_x(x) d\Gamma_\varsigma(\varsigma) - d\Upsilon_t(x, \varsigma)\Big) + (1 - \delta) d\Upsilon_t(x, \varsigma) \tag{11}$$

Note, the presence of a fixed cost of search generates an extensive margin of active search: workers only search if the net benefits of active search are positive. The fraction of non-employed workers engaged in active search $\check{\Gamma}_t^{ne}$ is given by

$$\check{\Gamma}_t^{ne} \equiv \int\limits_{X \times \mathcal{C}} \mathbb{I}\left\{s_{A,t}(x, \varsigma) > 0\right\} \cdot \left(\frac{d\Gamma_t^{ne}(x, \varsigma)}{1 - \Upsilon_t} \right) \tag{12}$$

given distributions of characteristics (x, ς) among the non-employed $\Gamma_t^{ne}(x, \varsigma)$ and the employment distribution $\Upsilon_t(x, \varsigma)$. In the next section, I derive the conditions describing the optimal level of active search, which in part determine the fraction of non-employed engaged in active search $\check{\Gamma}_t^{ne}$.

1.3. **The worker's problem.** Before writing down the problem of a worker, it is useful to establish some notation. Denote $\Lambda_{t,t+1} \equiv \mu_{t+1}/\mu_t$, where μ_t is the marginal utility of consumption at time t, equalized within the representative family via risk-sharing.[7] Consider the value of employment (in consumption good equivalents)

[7]The full derivation of the representative family's problem is given in Appendix B.

$W_t(x, \varsigma)$ of a worker with characteristics (x, ς) at time t, and let $U_t(x, \varsigma)$ denote the value of unemployment. Then, $W_t(x, \varsigma)$ can be written as

$$W_t(x,\varsigma) = w_t(x,\varsigma) + \mathbb{E}_t\Big\{\Lambda_{t,t+1} \cdot \Big[(1-\delta)\cdot W_{t+1}(x,\varsigma_{t+1}(\varsigma)) + \delta \cdot U_{t+1}(x,\varsigma_{t+1}(\varsigma))\Big] \tag{13}$$

where expectations are taken with respect to the law of motion for the aggregate state and the worker's fixed cost of search, $\varsigma_{t+1}(\varsigma)$.[8] The worker's value of employment depends on her wage $w_t(x, \varsigma)$, the probability of remaining employed δ, and the continuation values associated with unemployment and continued employment.

Then, the consumption-equivalent value of unemployment $U_t(x, \varsigma)$ for a worker with characteristics (x, ς) at time t is given by

$$U_t(x,\varsigma) = \max_{s_{A,t}(x,\varsigma)} \Bigg\{\frac{1}{\mu_t}\left(\psi - \varsigma \cdot \mathbb{I}\left\{s_{A,t}(x,\varsigma) > 0\right\} - \chi \cdot \frac{s_{A,t}^{1+\varkappa}(x,\varsigma)}{1+\varkappa}\right) \tag{14}$$
$$+ \mathbb{E}_t\Big\{\Lambda_{t,t+1} \cdot \Big[f_t(x,\varsigma)\cdot W_{t+1}(x,\varsigma_{t+1}(\varsigma)) + (1-f_t(x,\varsigma))\cdot U_{t+1}(x,\varsigma_{t+1}(\varsigma))\Big]$$

where expectations are taken with respect to the law of motion for the aggregate state and the worker's fixed cost of search, $\varsigma_{t+1}(\varsigma)$. Recall, $f_t(x, \varsigma)$ depends on $s_{A,t}(x, \varsigma)$, as described by equations (4) and (5):

$$f_t(x,\varsigma) = \Big(\omega \cdot s_{A,t}(x,\varsigma) + (1-\omega)\Big) \cdot f_t. \tag{15}$$

Also recall, ψ is the flow value of leisure, ς is fixed cost of search of the worker, χ is the scaling term for variable search costs, and $\varkappa$ determines the convexity of search costs.

Let $s_{A,t}(x, \varsigma)$ denote the policy function for active search associated with (14), and denote $H_t(x, \varsigma) \equiv W_t(x, \varsigma) - U_t(x, \varsigma)$. Then, $s_{A,t}(x, \varsigma) = s_t^{int}(x, \varsigma)$ if and only if

$$U_t(x,\varsigma)\Big|_{s_A = s_A^{int}} - U_t(x,\varsigma)\Big|_{s_A=0} \geq 0 \tag{16}$$

where

$$\frac{\chi}{\mu_t}\Big[s_{A,t}^{int}(x,\varsigma)\Big]^{\varkappa} = \mathbb{E}_t\Big\{\Lambda_{t,t+1}\cdot\omega\cdot f_t \cdot H_{t+1}\left(x,\varsigma_{t+1}(\varsigma)\right)\Big\} \tag{17}$$

with $s_{A,t}(x, \varsigma) = 0$ otherwise. Equations (16) and (17) describe the optimal search policy of a worker who sets the marginal cost of active search equal to its marginal benefit, conditional on the net benefit of active search exceeding zero; but otherwise does not actively search.

The model here is sufficiently general to be consistent with procyclical or countercyclical active search. Suppose that, during recessions, a lower aggregate job-finding probability f_t coincides with a higher marginal utility of consumption μ_t. Then, the problem of an unemployed worker (14) and the associated policy functions for

[8]The term $\varsigma_{t+1}(\varsigma)$ denotes the time $t + 1$ realization of the fixed cost of search, given that the time t realization is ς.

$s_{A,t}(x,\varsigma)$ described in (16) and (17) accommodate both substitution and income effects in the response to a drop in f_t, taking the wage as given.

By the substitution effect, a decline in the aggregate job-finding probability f_t reduces the marginal benefit of active search, as shown on the right-hand side of equation (17). The cost in forgone leisure of generating a given number of contacts increases, pushing the worker to engage in less active search.

By the income effect, an increase in the marginal utility of consumption μ_t reduces the consumption-equivalent marginal cost of active search, as shown on the left-hand side of equation (17).[9] Moreover, to the extent that the marginal utility of consumption μ_t is persistently high, the anticipated worker surplus will be similarly high from lower values of the consumption-equivalent flow value of unemployment ξ_t entering the value of unemployment $U_t(x,\varsigma)$ in equation (14), where

$$\xi_t(x,\varsigma) = \frac{1}{\mu_t}\left(\psi - \varsigma\cdot\mathbb{I}\left\{s_{A,t}(x,\varsigma)>0\right\} - \chi\cdot\frac{s_{A,t}^{1+\varkappa}(x,\varsigma)}{1+\varkappa}\right). \tag{18}$$

The combination of lower consumption-equivalent search costs and a higher expected surplus from employment induce greater active search effort during a recession.

Note, the consumption-equivalent flow value of unemployment $\xi_t(x,\varsigma)$ is equivalent to Chodorow-Reich and Karabarbounis's concept of the "opportunity cost of employment."[10] Chodorow-Reich and Karabarbounis (2016) carefully measure the aggregate opportunity cost of employment ξ_t and show it to be highly procyclical. This suggests that the income effect should dominate the substitution effect, so that a reduction in the aggregate job-finding probability f_t coinciding with an increase in the marginal utility of consumption μ_t should result in increased active search effort.

The empirical literature has shown that active search among non-employed workers is countercyclical along both the extensive and intensive margins, e.g. Osberg (1993), Shimer (2004), Elsby et al. (2015), Mukoyama et al. (2018), and Faberman and Kudlyak (2019), corroborating the notion that the procyclical aggregate opportunity cost of employment ξ_t should generate greater active search effort during a recession.

1.4. **The active-passive ratio in job-finding probabilities.** The rate at which active search increases a worker i's job-finding probability $f_{i,t}$ relative to the aggregate job-finding probability f_t is determined by the marginal efficiency of active search.[11]

In principle, the marginal efficiency of active search can be obtained from the data. For example, assume a Cobb-Douglass matching function with elasticity η and time-varying matching efficiency φ_t; and consider the translation of search efficiency into

[9]Note, the textbook presentation of the DMP model with endogenous search typically assumes that agents have linear utility over the consumption good, i.e., $\mu_t = 1$ for all t, thus implying unambiguously procyclical search activity by the unemployed. For example, see Pissarides (2000).

[10]There are two differences in the opportunity cost of employment here compared to Chodorow-Reich and Karabarbounis (2016): First, I consider an environment with an endogenously determined active search effort. Second, I abstract from UI payments.

[11]As I bring the model to the data, it will be useful to move between fully recursive notation to notation with time and individual subscripts. Unless stated otherwise, worker i is assumed to have characteristics (x,ς).

job-finding probabilities for a given individual i at time t,

$$f_{i,t} = (\omega \cdot s_{A,i,t} + (1-\omega)) \cdot \varphi_t \theta_t^{1-\eta} \tag{19}$$

If matching efficiency φ_t, market tightness θ_t, and active search $s_{A,i,t}$ are all observable, the parameters from (19) can be estimated by GMM. In particular efficient estimates of the marginal efficiencies of active and passive search ω and $1-\omega$ can be recovered.

In practice, however, not all of the relevant quantities are observed. For example, although progress is been made in quantifying the extent of match efficiency, it is still typically measured as a residual quantity, e.g. Barnichon and Figura (2015). Given that active search effort depends on (and thus unobserved match efficiency), one might concerned that estimates of ω may be inconsistent.

Rather than attempt to recover the marginal efficiencies of active and passive search time using an equation such as (19), I study the ratio of job-finding probabilities of the average active searcher relative to a passive searcher:

$$\begin{aligned} \frac{\bar{f}_{A,t}}{f_{P,t}} &= \frac{(\omega \cdot \bar{s}_{A,t} + (1-\omega)) \cdot \left(\frac{m_t(s_t,v_t)}{s_t}\right)}{(1-\omega) \cdot \left(\frac{m_t(s_t,v_t)}{s_t}\right)} \\ &= \left(\frac{\omega}{1-\omega}\right) \cdot \bar{s}_{A,t} + 1 \end{aligned} \tag{20}$$

where $\bar{s}_{A,t}$ is the average active search effort of the fraction $\check{\Gamma}_t^{ne}$ of the non-employed engaged in active search, i.e.

$$\bar{s}_{A,t} = \frac{\boldsymbol{s}_{A,t}}{\check{\Gamma}_t \cdot (1-\Upsilon_t)}. \tag{21}$$

According to equation (20), the ratio of the job-finding probability of the average active searcher relative to the job-finding probability of a purely-passive searcher depends only on the average search effort of the active searcher. Notably, all other variables are quasi-differenced from of the expression, including labor market tightness and match efficiency. Thus the average probabilistic gain from active over purely-passive search depends only on the average quantity of active search effort provided by active searchers.

Equation (20) has several notable features. First, although the model allows for heterogeneity in worker productivity, such heterogeneity maps exactly into the search decision of a worker. Thus, even if worker search varies as a function of underlying characteristics that are possibly unobserved to an analyst, the mapping of active search into search efficiency under random search implies that active search effort $s_{A,i,t}$ serves as a sufficient statistic for studying the idiosyncratic component to the worker specific job-finding probability $f_{i,t}$. Second, the active-passive ratio is independent of labor market tightness, and thus can be studied independently from a particular surplus splitting mechanism. Hence, cyclical variation in the active-passive ratio can be studied in isolation from the "unemployment volatility puzzle" à la Shimer (2005) and Hall (2005).

Finally, note that (20) is inconsistent with a model that accommodates a crowding-out of active search. To see why, consider a state of the economy in which only an infinitesimal mass of workers are engaged in active search. This would correspond to a situation where few non-employed workers were sending out job applications, and most non-employed workers were instead waiting to be contacted by an employer through professional networks or referrals. In the presence of crowding-out, the probability that an active searcher finds a job relative to a passive searcher should be higher in this context. However, for this to be reflected in the average probabilistic gain from active search, equation (20) would need to include terms describing the distribution of non-employed workers across active and passive search. Such terms are missing from the equation. Later, we will see that the absence of such terms is due to the assumption that active and passive search are perfect substitutes.

Given an empirical measures of active search effort and job-finding probabilities of the active an passive non-employed, (20) can be used not only to test against no-crowding-out of active search, but also to estimate the parameter ω that determines the marginal efficiencies of active and passive search, ω and $1-\omega$. Consider separate scenarios where active search varies along both the extensive and intensive margins à la Faberman et al. (2022), $s_{A,t}(x,\varsigma) \equiv s_{A,i,t} \in \mathbb{R}_+$; and where active search only varies along the extensive margin à la Krusell et al. (2017). Under each specification, subtract one and take logs to obtain,

$$\log\left(\frac{\bar{f}_{A,t}}{f_{P,t}} - 1\right) = \begin{cases} \log\left(\frac{\omega}{1-\omega}\right) + \log \bar{s}_{A,t} & \text{if } s_{A,i,t} \in \mathbb{R}_+ \\ \log\left(\frac{\omega}{1-\omega}\right) & \text{if } s_{A,i,t} \in \{0,1\} \end{cases} \tag{22}$$

For the case in which we consider both an extensive and intensive margin of active search, we estimate the log active-passive ratio (minus one) on the log average active search. If we cannot reject the null hypothesis that the coefficient on log average active search is one, we can recover the marginal efficiency of active search ω from the coefficient on the constant term. For the case in which we allow only an extensive margin of active search, our work is simpler: we simply check that the log active-passive ratio (minus one) is constant over time and uncorrelated with cyclically varying aggregates. If we cannot reject the null hypothesis that the active passive ratio is constant over time, we can recover the marginal efficiency of active search ω from the average value of the active-passive ratio in job-finding probabilities.

2. The empirics of the active-passive ratio in job-finding probabilities

This section uses merged monthly flows from the CPS over the period 1996 to 2019 to show that, (*i*) among the population of non-employed workers who are willing to work, the fraction of workers engaged in active search is countercyclical, (*ii*) conditional on engaging in active search, the intensity of active of search among the non-employed is also countercyclical, and finally, (*iii*) among non-employed workers willing to work, the ratio in job-finding probabilities of active versus passive searchers is procyclical.

The first two facts have been documented elsewhere in the literature (e.g., Mukoyama et al. 2018) and are consistent with the model of the previous section under a procyclical opportunity cost of employment, à la Chodorow-Reich and Karabarbounis

(2016). However, the combination of facts (*ii*) and (*iii*) — the recessionary rise in active search among the active non-employed, and the recessionary decline in the active-passive ratio in job-finding probabilities — is inconsistent with the three-state model of employment, unemployment, and non-participation developed in the previous section. Specifically, we can reject the restriction from the previous section that must hold in the absence of a crowding-out of active search.

I use CPS data from 1996 to 2019 to study job-finding outcomes of active and passive searchers. Subsequent to a 1995 redesign, the Current Population Survey monthly basic questionnaire introduce a series of questions that allow researchers to identify non-employed workers who are not searching but would be willing to accept a job.[12,13] Conditional on (*a*) not being employed or on temporary-layoff, and (*b*) being willing to accept work, non-employed workers are asked which in a series of nine methods of active search they are engaged in. Workers engaged in at least one method of active search meet the BLS definition of unemployed; whereas workers engaged in no active search methods are considered out of the labor force (although eligible for inclusion in broader measures of unemployment as "discouraged workers", such as U-4.)

I consider all unemployed workers not on temporary layoff as belonging to the active non-employed. I exclude workers on temporary layoff from the sample for a mix of theoretical and practical reasons: First, given that the majority of workers on temporary layoff are recalled to their previous job (Fujita and Moscarini, 2017), it would seem unreasonable to assume that reemployment outcomes of workers on temporary layoff are mediated by a same matching function process as for other workers in non-employment. Moreover, it is not clear whether such workers should be considered "active" or "passive": although workers on temporary layoff are re-employed at an even higher rate than the "active non-employed," they spend substantially less time on job-search related activities (Mukoyama et al., 2018). Finally, the CPS does not collect the additional information on search activity for workers on temporary layoff.

I classify a worker as belonging to the passive non-employed if the worker indicates that they would like to work, but that they are not engaged in active search; and furthermore, that their reason for not searching is for reasons due to the availability of work ("believes no work available," "couldn't find any work") rather than for reasons related to ability to supply labor (e.g. "family responsibilities," "ill-health, physical disability") or perceived discrimination (e.g. "employers think too young or too old"). If an individual provides no reason for not searching, I classify that individual as passive non-employed. If a worker is reports a willingness to work but does not meet the classification of passive non-employed, the worker is included among the out-of-the-labor-force (e.g., inactive). The classification generates five distinct labor force states: employment, temporary layoff, active non-employment, passive non-employment, and inactivity.

[12]1996 is the first year after the re-design for which it is possible to merge each of the twelve monthly basic files to study gross worker flows.

[13]Under the previous design, one could not distinguish among all non-employed respondents whether the respondent was willing or unwilling to accept work.

Table 1. Active and passive searchers in non-employment

	Active non-employed	Passive non-employed	$\frac{A\text{-}NE}{A\text{-}NE+P\text{-}NE}$	Avg. # of search methods
mean(x)	4.9	1.3	0.79	1.85
std(x)/std(Y)	11.0	5.7	1.50	2.65
corr(x, Y)	−0.89	−0.70	−0.75	−0.64

Table 2. *

Data from CPS, 1996-2019. Y indicates quarterly GDP. For second and third row, series are taken as (1) quarterly averages of seasonally adjusted monthly series, (2) logged, then (3) HP-filtered with smoothing parameter of 1600

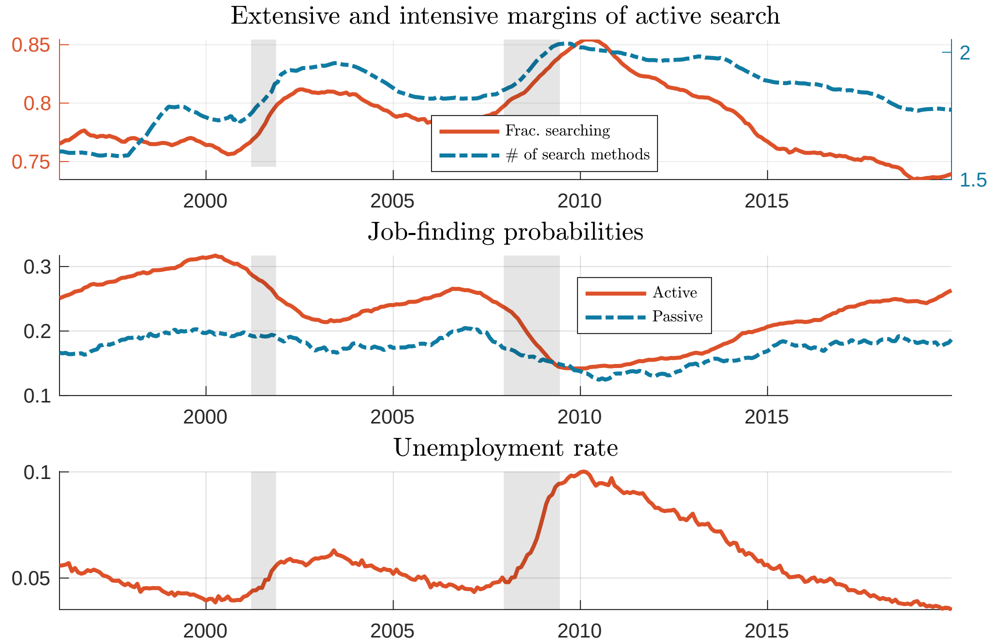
I use the number of active search methods as a measure of the intensive margin of active search. Such a measure has been used and validated elsewhere within the literature. Using data from Canada, For example, Osberg (1993) reports that the number of active search methods is countercyclical. Shimer (2004) reports the same for the United States from the post-1995 CPS. Mukoyama et al. (2018) study the extent to which an increase in number of search methods corresponds to an increase in time spent searching, finding that search is essentially linear in number of search methods, and they too find that search is countercyclical.[14] Evidence suggestive countercyclical search intensity has been found elsewhere; e.g., Faberman and Kudlyak (2019) show that workers in slacker labor markets apply to more jobs.

Table 1 gives descriptive statistics of the extensive and intensive margins of active search. The first two columns show the active and passive non-employment rates, computed as the number of active and passive non-employed divided by the number of workers in the labor force (plus the passive non-employed). The active non-employment rate is higher than the passive non-employment rate, at 4.9 percent to 1.3 percent. Although both are countercyclical, the active non-employment rate is nearly twice as volatile as the passive non-employment rate. Accordingly, the ratio of the active non-employed to the active plus passive non-employed — this paper's measure of the extensive margin of search — is mildly countercyclical, as documented in the third column of Table 1.[15] The top panel of Figure 1 plots the extensive and intensive margins of active search. Active search rapidly increases along both margins at the onset of a recession, and then gradually attenuates.

[14] See Figure 1, pg. 198. There is a slight break in linearity when number of search methods reaches five. In practice, however, very few active searchers are observed to be using a number of search methods within this range.

[15] The extensive margin of active search has been shown to be countercyclical using alternative measures of active and non-active search: see, for example, Mukoyama et al. (2018), who compute a countercyclical margin of search from the ratio of the unemployed to the sum of the unemployed and all workers out-of-the-labor-force.

FIGURE 1. Search and job-finding probabilities



Next, I describe the job-finding outcomes of workers in active and passive non-employment from longitudinally linked monthly data from the CPS. Following Shimer (2012) and Elsby et al. (2015) (among others), I use gross flows across labor market states to compute monthly transition probabilities that are adjusted for time aggregation bias and seasonally adjusted.

Table 3 provides summary statistics describing the resulting job-finding probabilities of the active and passive non-employed, as well as the active-passive ratio of job-finding probabilities. The job-finding probability of the active non-employed are roughly thirty percent larger than of the passive non-employed, at 0.23 to 0.17.[16] Thus, although the passive non-employed have lower monthly job-finding probabilities than the active non-employed, their average job-finding probabilities are substantially higher than than those of workers in inactivity, estimated here to be 0.04. Note that, although the job-finding probabilities of the active and passive non-employed are equally volatile, the job-finding probabilities of the passive non-employed are less correlated with aggregate GDP, by a factor of nearly three. Thus, the elasticity of the job-finding probability with respect to GDP is 7.37 ($= 8.67 \times 0.85$) for the active

[16]The job-finding probability of the active non-employed is comparable to other estimates of the job-finding probability from unemployment from procedures that acknowledge inactivity as a distinct labor market state, e.g. Krusell et al. (2017). In general, such estimates are lower than those produced from the procedure proposed by Shimer (2012), which effectively treats flows from unemployment to inactivity as flows from unemployment to employment.

Table 3. The active-passive ratio of job-finding probabilities

	A-$NE \to E$ probability	P-$NE \to E$ probability	A-P ratio
mean(x)	0.23	0.17	1/1.32
std(x)/std(Y)	8.67	8.87	9.53
corr(x,Y)	0.85	0.32	0.48

Table 4. *

Data from CPS, 1996-2019. A-NE and P-NE refer to active and passive non-employed, "A-P ratio" refers to active-passive ratio of job-finding probabilities, Y indicates quarterly GDP. For second and third row, series are taken as (1) quarterly averages of seasonally adjusted monthly series, (2) logged, then (3) HP-filtered with smoothing parameter of 1600

non-employed, versus just 2.84 (= 8.87 × 0.32) for the passive non-employed.[17] As such, the active-passive ratio in job-finding probabilities — the monthly job-finding probability of thep active non-employed divided by that of the passive non-employed — is estimated to be substantially procyclical, with an elasticity with respect to output of 4.56 (= 9.5 × 0.48).

Figure 1 shows the cyclical behavior of active search and the job-finding probabilities of the active and passive non-employed. At the onset of a recession, both the active search effort of the active non-employed and the fraction of active searchers among non-employed workers willing to accept work increase. Both quantities remain persistently high after the recession, returning to their pre-recession levels at roughly the same rate as the reduction in unemployment. Job-finding probabilities from active and passive non-employment are both procyclical, declining at the onset of a recession and showing a slow recovery. However, in spite of the greater search effort of the active non-employed during a recession, the extent of the recessionary decline in job-finding probabilities is drastically greater for the active non-employed: Indeed, the gap in the two job-finding rates is decreasing with the amount of active search effort. Such a pattern is consistent with a crowding-out of active search: where an increase in aggregate active search effort reduces the premium in job-finding probabilities associated with active search.

Recall, equation (20) offers a restriction relating the job-finding probabilities from active and passive non-employment to the average active search effort of the active non-employed, formulated from a generic three-state DMP model where active and

[17]This paper focuses on the passive non-employed as a narrow segment of the non-participant who are more comparable to workers in unemployment. However, the greater cyclicality of job-finding rates of active searchers relative to passive searchers extends more broadly to all non-participants: e.g., see Table 3 of Krusell et al. (2017). To my knowledge, however, the paper here is the first to explore the implications of this regularity for business cycle analysis. For example, although Krusell et al. document that job-finding rates of the unemployed are more cyclical than for non-participants, they calibrate their model so that the two groups of non-employed workers have equally cyclical job-finding probabilities— see pg. 3464, second paragraph.

FIGURE 2. The active-passive ratio and average search effort

passive search are taken as perfect substitutes. Notably, the fraction of the non-employed engaged in active search does not enter the restriction; and the restriction predicts that the active-passive ratio in job-finding probabilities should be increasing in the active search effort of the active non-employed. Figure 2 plots the active-passive ratio in job-finding probabilities against the average quantity of active search among the active non-employed, both taken from the data: the apparent relationship is strikingly negative. The empirical pattern shown in Figure 2 visually foreshadows that the theoretical restriction summarized in equation (20) will be easily rejected.

2.1. Estimating the elasticity of the active-passive ratio in job-finding probabilities.

Here, I test the restrictions from (22). Conditional on being unable to reject the restrictions in the data, the marginal efficiencies of active and passive search can be easily recovered from the data.

I estimate the following equation by OLS:

$$\log\left(\frac{\bar{f}_{A,t}}{\bar{f}_{P,t}} - 1\right) = \beta_0 + \beta_\# \cdot \log \bar{s}_{A,t} + \epsilon_t, \tag{23}$$

where ϵ_t is an *iid* error term. Under a theory admitting operative extensive and intensive margins of active search à la Faberman et al. (2022), the estimated coefficient $\beta_\#$ should be equal to one. Under a theory admitting only an extensive margin of search à la Krusell et al. (2017), cyclical variation in $\bar{s}_{A,t}$ is uninformative about

TABLE 5. Elasticity of active-passive ratio in job-finding probabilities

Dependent variable: Log active-passive ratio in in job-finding probabilities (minus one)			
	(1)	(2)	(3)
Log # of search methods	-7.609^{***}	-4.857^{***}	-3.006^{***}
	(0.8975)	(0.3933)	(0.1487)
Time trend	$-8.0\text{e-}4^{*}$	$-4.3\text{e-}4^{**}$	$-7.9\text{e-}5$
	(4.5e-4)	(2.0e-4)	(7.9e-5)
Constant	4.004^{***}	2.808^{***}	3.228^{***}
	(0.4755)	(0.2180)	(0.0947)
$p(\beta_{\#} = 1)$	0.000	0.000	0.000
N	279	288	288
Passive searchers:	Want job (discouraged)	Want job (all)	Nilf

CPS, 1996-2019

variation in the active-passive ratio; but to the extent that $\bar{s}_{A,t}$ is correlated with other labor market aggregates, the coefficient $\beta_{\#}$ should be zero.

The first column of Table 5 reports estimates from the benchmark regression specification. The point estimate for the elasticity is -5.85. Thus, a one percent increase in average active search effort is associated with an almost six percent reduction in the job-finding probability of the active non-employed relative to the passive non-employed. Instead, the theory predicts an elasticity of one, under intensive and extensive margins of active search; or an elasticity of zero, with only an extensive margin of active search.

Hence, a crucial theoretical restriction implied under three-state DMP models can be easily rejected. Whereas these models imply that an increase in average search effort raises the premium in average job-finding probabilities of the active non-employed over the passive non-employed, we see the reverse in the data. Given the summary statistics provided in the previous section, the rejection of the theoretical restriction should come as no surprise: given the procyclical active-passive ratio in job-finding probabilities and countercyclical active search, the former cannot possibly have a unit elasticity in the latter, ruling out the possibility of a unit elasticity under intensive and extensive margins of endogenous active search; or an elasticity of zero, under only an extensive margin of active search.

2.2. **Robustness.** One might then reasonably ask: absent relaxing the assumption of perfect substitution between active and passive search to allow for a crowding-out of active search, what are the minimal changes to the three state DMP model structure necessary for it to be consistent with the data? In this section, I consider the robustness of my estimates to the inclusion of additional aggregate variables, the allowance of an independent stochastic process for the marginal efficiency of active search, controls for duration dependence, and cyclical heterogeneity in the composition of active and passive searchers. To the extent that any one of these candidate explanations can account for the negative elasticity of the active-passive ratio with respect to search effort, they might suggest some minimal alteration that might allow a three-state DMP model with perfect substitution of aggregate and passive search to be consistent with the data. As will be seen, however, none of these additional considerations can explain account empirically for the declining active-passive ratio of job-finding probabilities in the average quantity of active search.

2.2.1. *Additional aggregate variables.* As a first pass, one might consider adding additional aggregate variables to the regression equation, (23). The introduction of a variable z_t to the regression equation can be understood as a slight modification to (4) and the equations that follow, replacing (4) with

$$s_{i,t} = \omega \cdot e^{z_t} \cdot s_{A,i,t} + (1-\omega) \cdot 1 \tag{24}$$

and (6) with

$$\boldsymbol{s}_t = \omega \cdot e^{z_t} \cdot \boldsymbol{s}_{A,t} + (1-\omega) \cdot \boldsymbol{s}_{P,t} \tag{25}$$

Then, the active passive-ratio implied under the model changes to

$$\log\left(\frac{\bar{f}_{A,t}}{f_{P,t}} - 1\right) = \begin{cases} z_t + \log\left(\frac{\omega}{1-\omega}\right) + \log \bar{s}_{A,t} & \text{if } s_{A,i,t} \in \mathbb{R}_+ \\ z_t + \log\left(\frac{1}{1-\omega}\right) & \text{if } s_{A,i,t} \in \{0,1\} \end{cases} \tag{26}$$

with resulting regression equation

$$\log\left(\frac{\bar{f}_{A,t}}{f_{P,t}} - 1\right) = \beta_0 + \beta_z \cdot z_t + \beta_\# \cdot \log \bar{s}_{A,t} + \epsilon_t \tag{27}$$

The restriction imposed by the model on the coefficient $\beta_\#$ is unchanged; but now, we can control for possibly confounding aggregate variation.

A clear candidate for such an aggregate variable z_t would be a time-trend. Figure 2 shows a downward trend in the active-passive ratio, and a slight upward trend in active search effort. Ignoring these trends could bias our estimates; and it is standard in the matching function literature to control for secular trends, e.g. Blanchard and Diamond (1989). The second column of Table 2 show estimates of $\beta_\#$ where we allow for a time trend: the estimated coefficient is still negative, large in magnitude, and precisely estimated.

Another candidate to include in z_t is a cyclical indicator, such as the unemployment rate. If the addition of such a cyclical indicator allows the theoretical restriction to hold, one might conclude that the negative elasticity is not indicative of some fundamental relation of the active-passive ratio to active search effort, but rather follows from a mechanical cyclical relationship. Thus, the third column of Table 5 shows estimates from a regression including a time trend and the unemployment rate as controls. The estimated elasticity $\beta_\#$ is still large, negative, and precisely estimated.

2.2.2. *An independent stochastic process for the marginal efficiency of active search.* The introduction of a time trend to the estimation equation, equation (23) — and then a time trend and the unemployment rate — allows for changes in the active-passive ratio that move with the realization of the aggregate state. Thus, the active-passive ratio is allowed update with innovations to the aggregate state independently from changes in average active search $\bar{s}_{A,t}$.

However, this does not rule out a separate concern of random fluctuations in the marginal efficiency of active search, i.e.

$$\omega_t = \omega + v_t \tag{28}$$

where v_t is distributed *iid*. While this is a possibility without clear economic motivation, it is still important to consider: recall from equation (17) that the quantity of active search provided is increasing in its marginal efficiency. Thus, in independent stochastic process for the marginal efficiency of active search such as that given in (28) would generate a positive covariance between $\bar{s}_{A,t}$ and ω_t that would bias our estimates. As I show below, however, such a bias would add a positive bias to the estimated coefficient $\beta_{\#}$, and thus could not account for the negative estimates that we recover in the previous section.

To be precise, assume that the true data generating process is described by

$$\log\left(\frac{\bar{f}_{A,t}}{f_{P,t}} - 1\right) = \log\left(\frac{\omega_t}{1-\omega_t}\right) + 1 \cdot \log \bar{s}_{A,t} + \epsilon_t \tag{29}$$

where ω_t is defined as (28). Then, the estimate for $\beta_{\#}$ would be given by

$$\widehat{\beta}_{\#} = \frac{\operatorname{cov}\left(\log \bar{s}_{A,t}, \log\left(\frac{\bar{f}_{A,t}}{f_{P,t}} - 1\right)\right)}{\operatorname{var}\left(\log \bar{s}_{A,t}\right)} \tag{30}$$

From (29),

$$\operatorname{cov}\left(\log \bar{s}_{A,t}, \log\left(\frac{\bar{f}_{A,t}}{f_{P,t}} - 1\right)\right) = \operatorname{cov}\left(\log \bar{s}_{A,t}, \log\left(\frac{\omega_t}{1-\omega_t}\right)\right) + \operatorname{cov}\left(\log \bar{s}_{A,t}, 1 \cdot \log \bar{s}_{A,t}\right)$$

But, given that $\log\left(\omega_t/(1-\omega_t)\right)$ is increasing in ω_t, and that $\bar{s}_{A,t}$ is increasing in ω_t, we know that

$$\operatorname{cov}\left(\log \bar{s}_{A,t}, \log\left(\omega_t/(1-\omega_t)\right)\right) > 0.$$

Returning to equation (30),

$$\begin{aligned} \widehat{\beta}_{\#} &= 1 + \frac{\operatorname{cov}\left(\log \bar{s}_{A,t}, \log\left(\frac{\omega_t}{1-\omega_t}\right)\right)}{\operatorname{var}\left(\log \bar{s}_{A,t}\right)} \\ &> 1 \end{aligned} \tag{31}$$

Hence, in the presence of an independent stochastic process for the marginal efficiency of active search ω_t, the estimates of Table 5 are upward biased. Thus, such a process appears to be an unlikely candidate explanation for the negative elasticity estimated from the data.

2.2.3. *Duration dependence.* It has been long established that the exit rate from unemployment is declining in the duration of nonemployment, e.g. van den Berg and van Ours (1996) and Kroft et al. (2013).

This could be a concern if we assume that the source of such duration dependence can be attributed to a marginal efficiency of active search that is declining over a non-employment spell. Given an average duration of non-employment that is increasing during a recession, such form of duration dependence could add a source of bias to the regression that could work to either increase or decrease the elasticity of the active-passive ratio with respect to active search effort.

If non-employed workers with long durations of non-employment are more likely to search, the job-finding probability from active non-employment would become more cyclical: the increased presence of workers with a low marginal efficiency of active search in the pool of non-employed workers would further reduce the job-finding probability from active non-employment during a recession, introducing a negative bias to our estimate of $\beta_{\#}$. This type of bias could explain the large and negative estimates of $\beta_{\#}$ from Section 2.1.

On the other hand, if non-employed workers with long durations of non-employment are less likely to engage in active search search, the job-finding probability from passive non-employment would become more cyclical and thus further decrease during a recession. But because the job-finding probability of passive searchers is in the denominator of the active-passive ratio that enters the restriction, such an effect would decrease the overall cyclicality of the active-passive ratio, lending an upward bias to our estimate of $\beta_{\#}$. This type of bias would not explain the large and negative estimates of $\beta_{\#}$, but instead imply that the estimates of 2.1 are large and negative in spite of a positive bias.

We are restricted from considering both sources of duration dependence in the data: the CPS only collects information on duration of unemployment for workers engaged in active search. Thus, while we can compute the job-finding probability and search effort of the active non-employed by their duration of nonemployment, we are unable to compute a job-finding probability for the passive non-employed similarly. However, we can construct a active-passive ratio in the job-finding probabilities that is semi-corrected for duration dependence, where the job-finding probability of the numerator is restricted to workers with low durations of non-employment; but where the job-finding probability of denominator is not. Given that duration dependence enhances the procyclicality of the active-passive ratio through the numerator; and moderates the procyclicality through the nominator; such a semi-corrected active-passive ratio will over-correct for duration dependence. Hence, if we re-estimate (22) with the semi-corrected measure, the recovered elasticity provides an upper bound for the influence of duration dependence on the active-passive ratio in job-finding probabilities. Should we still find that the relevant elasticity is negative, we can therefore conclude that duration dependence is not alone responsible for the rejected restriction.

I restrict the sample of active searchers to consist of those reporting an unemployment duration of less than or equal to 14 weeks. This changes the composition of the

TABLE 6. Elasticity of active-passive ratio in job-finding probabilities: controls for duration dependence

Dependent variable: Log active-passive ratio in in job-finding probabilities (minus one)			
	(1)	(2)	(3)
Log # of search methods	-1.717^{***}	-1.581^{***}	-1.748^{***}
	(0.3827)	(0.2195)	(0.1066)
Time trend	−1.6e-4	4.5e-5	2.0e-4***
	(2.6e-4)	(1.5e-4)	(7.3e-5)
Constant	0.832^{***}	1.073^{***}	2.595^{***}
	(0.2234)	(0.1281)	(0.0623)
$p(\beta_{\#} = 1)$	0.000	0.000	0.000
N	288	288	288
Passive searchers:	Want job (discouraged)	Want job (all)	Nilf

CPS, 1996-2019

sample affects the numerator of the active-passive ratio in job-finding probabilities, as well as the average search effort of the active non-employed. However, the denominator of the active-passive ratio in job-finding probabilities is unaltered. Estimates from the semi-corrected measure are given in Table 6. While the point estimates of the coefficients drop, the precision of the estimates increase. The recovered elasticities, with and without controls, all fall uniformly below -2.25. Notably, where the estimated elasticity previously fell by about half when the unemployment rate was added as a control in the original estimates from Table 5, the estimated elasticity hardly changes here.

Thus, the estimated coefficients suggest that duration dependence alone cannot explain the negative elasticity estimated under the benchmark regressions from Table 5.

2.2.4. *Cyclical composition.* The theory presented in Section 1 prescribes a role for cyclical composition in the distribution of non-employed workers across active and purely-passive search through the endogenous choice of active search intensity, $s_{A,i,t}$ as a function of unobserved characteristics. As such, there is no worry per sè that the theoretical restriction might not be robust to cyclical changes in the composition of workers in active versus passive non-employment.

However, if the marginal efficiency of active search varies across groups j, and the representation a particular group within active or passive non-employment changes over the business cycle, this could bias our estimates. For example, assume that a group j with a particularly low marginal efficiency of active search ω_j assumes a larger fraction of the active non-employed during recessions. Then, the negative elasticity of the active-passive ratio in average search effort could be purely due to the countercyclically increasing representation of workers with a low marginal efficiency of active search within the pool of the non-employed.

To consider possible implications of cyclical composition for my estimates, I follow Shimer (2012), Elsby et al. (2015), Krusell et al. (2017), and Mueller (2017) by isolating the role of cyclical composition using a “shift-share” estimator. First, I identify various dimensions of heterogeneity observable across individuals in active and passive non-employment, e.g., age or education. Next, I compute job-finding probabilities for the active and passive non-employed (along with active search) within groups. Then, I construct weighted average of each measure with time invariant weights. For example, indexing groups by j, the re-weighted active job-finding probability $\tilde{\bar{f}}_{A,t}$ is

$$\tilde{\bar{f}}_{A,t} = \sum_j \bar{\pi}_{A,j} \bar{f}_{A,j,t} \tag{32}$$

where $\bar{\pi}_{A,j}$ is the average fraction of workers of type j in the population of active searchers over time, and $\bar{f}_{A,j,t}$ is the job-finding probability for the active non-employed of subgroup j at time t. We can similarly construct a re-weighted average active search effort $\tilde{\bar{s}}_{A,j,t}$, and use weights $\bar{\pi}_{P,j}$ to construct the re-weighted passive job-finding probability $\tilde{\bar{f}}_{P,t}$.

In the final step, I re-estimate (22) with the re-weighted job-finding probabilities and measure of active search effort. The estimates from the re-weighted data provide a test of whether the estimates from the true data are driven by cyclical composition: Should the estimated elasticity from Table 5 reflect a greater recessionary presence of workers from a type j characterized a lower marginal efficiency of active search among active searchers, for example, the fixed weights $\bar{\pi}_{A,j}$ and $\bar{\pi}_{P,j}$ mitigate the potential for bias. Hence, to the extent that the estimated elasticities from the re-weighted variables change sufficiently that we can no longer reject the proportional job-finding probability restriction, we can conclude that the previous estimates are driven by cyclical heterogeneity in the composition of active and passive non-employment.

A limitation of this exercise is one of statistical power: for each subgroup, we must construct a job-finding probability from CPS respondents matched across monthly surveys. If we consider heterogeneity binned into groups that are too sparse, the monthly job-finding probabilities for a subgroup j, $\bar{f}_{A,j,t}$, will be too noisy to produce reliable estimates.[18] Thus, following Shimer (2012), Elsby et al. (2015), Krusell et al. (2017), and Mueller (2017), I consider several dimensions of heterogeneity across separate decompositions: gender, race, age, marital status by gender, education, and region. A list of subgroups with weights and averages of the re-weighted data series are given in Table **??**.

[18] An alternative approach is to study the relation of job-finding probabilities across the active and passive non-employed from micro-level data using a linear probability or probit model. I present such an analysis in Appendix A.

Table 7. Elasticity of active-passive ratio in job-finding probabilities: cyclical composition

Dependent variable: Log active-passive ratio in in job-finding probabilities (minus one)						
	None	Time trend	Time trend + unempl. rate	None	Time trend	Time trend + unempl. rate
	1. Gender			*4. Marital status (by gender)*		
Log # of search methods	−6.447***	−5.760***	−3.004***	−6.126***	−5.465***	−2.520***
	(0.9040)	(1.0593)	(0.9476)	(0.7903)	(0.9173)	(0.8010)
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000	0.000	0.000	0.000
N	265	266	266	265	265	265
	2. Race			*5. Education*		
Log # of search methods	−6.150***	−5.355***	−3.439***	−5.744***	−4.961***	−3.458***
	(0.7947)	(0.9732)	(1.0255)	(0.9564)	(1.0153)	(1.1548)
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000	0.000	0.000	0.000
N	265	265	265	223	223	223
	3. Age			*6. Region*		
Log # of search methods	−6.211***	−4.998***	−2.117***	−5.870***	−4.910***	−2.659***
	(0.8260)	(0.9519)	(0.7850)	(0.8166)	(0.9365)	(0.9044)
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000	0.000	0.000	0.000
N	267	267	267	265	265	265

Note:

For each dimension of heterogeneity, I re-estimate the regression equation 23, with and without controls. Results are given in Table 7. In every instance, the estimated elasticity is large in magnitude and negative, and we can easily reject the null hypotheses that the estimate is one or zero. If anything, the estimated elasticities from Table 7 are larger in magnitude than the elasticities from the benchmark specifications of Table 5. The estimated elasticities are thus consistent with an interpretation that cyclical heterogeneity in the composition of the active and passive non-employed dampens the estimated negative elasticity of the active-passive ratio to active search effort.

Notably, the findings above are consistent with those of Mueller (2017), who studies the cyclicality of separation and job-finding rates for workers in unemployment across groups defined by various characteristics. While Mueller finds strong evidence for differential cyclicality for separation rates within groups, he finds that the cyclicality of job-finding rates is essentially uniform across groups. Such findings also hold for groups sorted by residual wages, which is taken to imply a minimal role for unobserved heterogeneity in explaining the cyclicality of job-finding rates.

In Section 1 of the paper, I develop a restriction that holds in the absence of the crowding-out of active search under a three-state DMP model of unemployment, employment, and non-participation. In this section, I show the restriction to be rejected. This result is robust to systematic variation in the relation of the active-passive ratio to active search effort over time and the business cycle; the allowance of an independent stochastic process governing the evolution of the marginal efficiency of active search; the allowance for duration dependence in job-finding rates; and controls for cyclical composition in the pool of active and passive non-employed. Going forward, I explore a less restrictive setting allowing for imperfect substitution of active and passive search, and thus a crowding-out effect of active search.

3. A general CES search aggregator

In the previous section, we robustly rejected the restriction developed Section 1 that must hold in the absence of a crowding-out of active search under a three-state model of employment, unemployment, and inactivity. Here, I consider a simple modification of that model, where a crowding-out of active search can be accommodated through a diminishing marginal efficiency of active search. This requires relaxing the assumption that active and passive search enter the matching function as perfect substitutes, instead allowing the elasticity of substitution between active and passive search to be estimated from the data.

3.1. **Revisiting the theory.** Return to the model of Section 1; but now consider a CES aggregator over active and passive search, where aggregate search efficiency s_t is defined as

$$\boldsymbol{s}_t = \Big(\omega \cdot (z_t \cdot \boldsymbol{s}_{A,t})^{\rho} + (1-\omega) \cdot \boldsymbol{s}_{P,t}^{\rho}\Big)^{\frac{1}{\rho}} \tag{33}$$

with $\rho \leq 1$. We take z_t as a flexible time-series process that influences the relative weight of active search in determining total search efficiency. Aggregate active and passive search, $\boldsymbol{s}_{A,t}$ and $\boldsymbol{s}_{P,t}$, are defined exactly as in equations (7) and (8). The marginal efficiency of active and passive search can be expressed as follows:

$$ME_{A,t} = \frac{\partial \boldsymbol{s}_t}{\partial \boldsymbol{s}_{A,t}} = \omega \cdot z_t^{\rho} \cdot \left(\frac{\boldsymbol{s}_t}{\boldsymbol{s}_{A,t}}\right)^{1-\rho}, \quad ME_{P,t} = \frac{\partial \boldsymbol{s}_t}{\partial \boldsymbol{s}_{P,t}} = (1-\omega) \cdot \left(\frac{\boldsymbol{s}_t}{\boldsymbol{s}_{P,t}}\right)^{1-\rho} \tag{34}$$

Note, the equations above are the same as the corresponding equations of Section 1 when $\rho = 1$ and $z_t = 1$.

To proceed further, we must derive expressions for the search efficiency input of a worker i. Invoking linear homogeneity of the search aggregator, the search efficiency $s_{i,t}$ of a worker i at time t can be written as

$$s_{i,t} = ME_{A,t} \cdot s_{A,i,t} + ME_{P,t} \tag{35}$$

As in equation (5), an individual's search efficiency $s_{i,t}$ enters multiplicatively into the equation for the individual's job-finding probability:

$$f_{i,t} = (ME_{A,t} \cdot s_{A,i,t} + ME_{P,t}) \cdot \left(\frac{m_t(\boldsymbol{s}_t, \upsilon_t)}{\boldsymbol{s}_t}\right) \tag{36}$$

Under perfect substitutes (i.e. $\rho = 1$) and with $z_t = 1$, the marginal efficiencies of active and passive search simplify to constants. Thus, the job-finding probability $f_{i,t}$ is multiplicatively separable into two components: one which depends on individual search effort but no aggregate components; and another that depends only on aggregate components. When $\rho < 1$, however, the marginal efficiencies of active and passive search entering the former term now depend on the composition of aggregate search.

We can use equation (36) and solve for the marginal efficiency of active and passive search $ME_{A,t}$ and $ME_{P,t}$ to write out the implied active-passive ratio in job-finding probabilities as a function of parameters, the fraction of non-employed engaged in active search $\check{\Gamma}_t$, and the average active search effort of the active non-employed $\bar{s}_{A,t}$:

$$\frac{\bar{f}_{A,t}}{f_{P,t}} = z_t^{\rho} \left(\frac{\omega}{1-\omega}\right) \left(\frac{1}{\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}}\right)^{1-\rho} \cdot \bar{s}_{A,t} + 1 \tag{37}$$

The term pre-multiplying the final $\bar{s}_{A,t}$ in equation (37) is the ratio of marginal efficiencies of active and passive search. Given the properties of a CES aggregator, we can see that the relative returns to active search should be uniformly positive, but decreasing in the term $\check{\Gamma}^{ne} \cdot \bar{s}_{A,t}$— except under the knife-edge condition where $\rho = 1$, i.e. under perfect substitutes.

Subtracting one and taking natural logarithms, we obtain

$$\log\left(\frac{\bar{f}_{A,t}}{f_{P,t}} - 1\right) = \rho \log z_t + \log\left(\frac{\omega}{1-\omega}\right) + (\rho - 1)\log \check{\Gamma}_t^{ne} + \rho \log \bar{s}_{A,t} \tag{38}$$

The log active-passive ratio (minus one) under the unrestricted CES estimator is similar to that of Section 1, but with two key differences. First, rather than entering with a coefficient of unity, the term $\log \bar{s}_{A,t}$ carries a coefficient that can take on negative values, equal to ρ. Second, we see an additional variable, $\check{\Gamma}_t^{ne}$, reflecting the dependence of the active-passive ratio on the fraction of non-employed engaged in active search. Furthermore, the coefficient on this variable is linear in ρ. As will be exploited in the next section, this provides an over-identifying restriction that can be used to statistically reject the model structure.

Next, I estimate the parameters of the CES aggregator from equation (38). I also exploit the testable restriction of equation (38) that allows for rejection of the aggregator.

3.2. **Estimating the elasticity of substitution between active and passive search.** I return to the data to estimate the coefficient for equation (38). The estimation equation is

$$\log\left(\frac{\bar{f}_{A,t}}{f_{P,t}}-1\right)=\beta_0+\beta_{\#}\cdot\log\bar{s}_{A,t}+\beta_{Frac}\cdot\log\check{\Gamma}_t^{ne}+\beta_z\cdot\log z_t+\epsilon_t \tag{39}$$

I consider the same two parametric forms for z_t introduced in Section 2: a time trend, e.g. $z_t=\exp(\phi_t\cdot t)$, and a time trend and the unemployment rate, e.g. $z_t=\exp(\phi_t\cdot t+\phi_u\cdot UR_t)$. For set of controls, I estimate three forms of equation (39). In the first, I leave the regression equation unrestricted, and I test the null hypothesis that $\beta_{\#}=\beta_{Frac}+1$. In the second form, I impose the restriction $\beta_{\#}=\beta_{Frac}+1$; conditional on not being able to reject the restriction from the first specification, this second specification recovers the parameters of the unrestricted CES ρ and ω and allows for the calculation of the elasticity of substitution $1/(1-\rho)$. In the third and final form, I consider the case where there is no intensive margin of search, and thus I omit the variable from the regression. For this final case, I can once again recover the parameters of the CES aggregator.

The first three columns of Table 8 presents estimates of the parameters of the search aggregator with a time trend, with the first column offering estimates of coefficients of the unrestricted regression. The point estimates of β_{Frac} and $\beta_{\#}$ are qualitatively consistent with the restriction summarized in equation (38). We cannot reject the null hypothesis that the restriction $\beta_{\#}=\beta_{Frac}+1$ holds, with a p-value of 0.719. Thus, we are free to re-estimate (39) imposing the model-implied restriction.

Estimates from the restricted regression are given in the second column of Table 8. We reject the null hypothesis that $\rho=1$, i.e. that active and passive search are perfect substitutes, with a p-value that is essentially zero. Hence, from the first regression, we are unable to reject that active and passive search enter the matching function bundled through a CES aggregator; but from the second regression, we can reject the assumption of perfect substitutes that is pervasive from much of the literature on three-state models. The implied values of ρ and ω are given as -3.12 and 0.73. Hence, the elasticity of substitution $1/(1-\rho)$ equals about one-fourth, with the estimate of ω indicating a substantial weight put on aggregate active search within the CES aggregator.

The third column of Table 8 presents estimates under the restriction $\beta_{\#}=0$, corresponding to an economic environment where there is no operative intensive margin of active search. Once again, we are able to reject the null hypothesis that $\rho=1$,

Table 8. Estimates of the active-passive ratio under an unrestricted CES aggregator

	(1)	(2)	(3)	(4)	(5)	(6)
β_{Frac}	-6.029^{***}	-5.374^{***}	-10.468^{***}	-2.771^{***}	-2.460^{***}	-3.295^{***}
	(1.9596)	(0.5413)	(1.2716)	(0.4071)	(0.1465)	(0.2374)
$\beta_{\#}$	-3.905^{***}	-4.374^{***}	—	-0.950^{*}	-1.460^{***}	—
	(1.3223)	(0.5413)		(0.5268)	(0.1465)	
β_0	1.041	1.393^{***}	-1.679^{***}	−0.436	−0.040	-1.147^{***}
	(0.9789)	(0.2520)	(0.3452)	(0.4291)	(0.0933)	(0.1553)
Passive searchers:	Want job, discouraged			Want job, all		
Constrain $\beta_{\text{Frac}} + 1 = \beta_{\#}$?	No	Yes	—	No	Yes	—
F-test	$p(\beta_{\text{Frac}} + 1 = \beta_{\#})$	$p(\rho = 1)$	$p(\rho = 1)$	$p(\beta_{\text{Frac}} + 1 = \beta_{\#})$	$p(\rho = 1)$	$p(\rho = 1)$
	= 0.716	= 0.000	= 0.000	= 0.358	= 0.000	= 0.000
N	279	279	279	288	288	288
Implied ρ	—	−4.374	−11.468	—	−1.460	−4.295
Implied ω		0.801	0.157		0.490	0.241

Note: CPS, 1996-20019

and we obtain estimates of ρ and ω of -9.14 and 0.259, indicating an elasticity of substitution equal to about one-tenth. The estimates of ρ and ω from the second and third columns indicate that a model lacking an intensive margin of active search requires a smaller elasticity of substitution and a lower weight on active search to be consistent with the data.

The fourth through sixth columns of Table 8 repeat the regressions of the first through third, but adding the unemployment rate as a control. This corresponds to a parametric form whereby the marginal efficiency of active search to evolves over the business cycle as a deterministic function of the unemployment rate. From the standpoint of an economic theory, such a parameterization may be seen as unappealing and rather ad-hoc. However, the introduction of this additional control allows one to assess the extent to which the lower elasticity of substitution estimated in columns two and three might be driven by business cycle variation unrelated to the relative aggregate quantities of active and passive search. The estimates of these columns are noisier, as might be expected; but we still recover estimates of ρ indicating an elasticity of substitution substantially lower than one. Hence, we can conclude that our estimates are not driven by ad-hoc business cycle variation.

This section has shown that we can accommodate an active-passive ratio in job-finding probabilities that is decreasing in average search if we are willing to consider active and passive search as imperfect substitutes. We are unable to reject the general structure that allows for such a possibility, and our resulting estimates indicating an elasticity of function that is substantially below one, implying a sharply diminishing marginal efficiency of active search. For a worker, this is consistent with a premium from active search that is greater when fewer other workers are engaged in active search, consistent with crowding-out. For the firm, the estimates of active and passive search entering the matching function as complements is consistent with a recruiting process that favors a relatively stable ratio of outside applicants to applicants that are known to the firm.[19]

4. Applications

The estimates of the previous section imply a diminishing marginal efficiency of active search. Here, I illustrate how these estimates generate novel insights for unemployment dynamics over the business cycle and the role of policy.

First, I study the implications of my estimates for unemployment and job-finding dynamics at business cycle frequencies. Recall, our estimates indicated not only a diminishing marginal efficiency of active search, but also an elasticity of substitution between active and passive search less than one. I provide a simple representation theorem to generate intuition for what these estimates mean. Then, I apply results from the theorem to provide a structural interpretation for the convergence of job-finding probabilities of the active an passive non-employed during a recession.

[19]Recall, at the limit where active and passive search enter the matching function as perfect complements, the implied aggregator is Leontief.

Second, I revisit the Bailey-Chetty formula for optimal unemployment insurance, which describes how optimal unemployment insurance benefits (UI) are limited by the disincentive effect of UI on active search, as described by the micro-elasticity of unemployment with respect to UI replacement rates. While this micro-elasticity is taken to be fixed in much of the existing literature, I show that it is proportional to the elasticity of total search efficiency with respect to active search. The estimates of this latter object of the previous section imply that the micro-elasticity is declining during recessions, offering a novel rationale for increasing UI benefits during a recession.

Finally, I return to the model developed in Sections 1 and 3, and I consider the problem of a social planner who seeks to maximize social welfare. I show that, given a diminishing marginal efficiency of active search, the social planner prescribes less active search the greater the dispersion of heterogeneity in the marginal social value of an employed worker. Intuitively, when the marginal efficiency of active search is decreasing in the total quantity of active search, the planner does not want the active search of low social value workers to overly reduce the marginal efficiency of active search and crowd out the active search of high social value workers. Such an efficiency channel is only present when active and passive search are imperfect substitutes.

4.1. **Understanding the procyclical active-passive ratio in job-finding probabilities.** The paper has established that active and passive search enter the matching function with limited substitutability. However, the notion of an unrestricted CES aggregator over different types of search entering a matching function might seem somewhat unfamiliar. The following proposition offers an equivalent and more intuitive representation that lends itself more easily to business cycle analysis.

Proposition 1 (Equivalence result)**.** *A CES aggregator over active and passive search entering a single matching function is equivalent to active and passive search entering separate matching functions, but weighted by their marginal efficiencies:*

$$m_t(\mathbf{s}_t, \upsilon_t) = m_t(ME_{A,t} \cdot \mathbf{s}_{A,t}, \alpha_t \cdot \upsilon_t) + m_t\left(ME_{P,t} \cdot \mathbf{s}_{P,t}, (1-\alpha_t)\cdot \upsilon_t\right), \tag{40}$$

where α_t is the active search share of vacancies, i.e. the fraction of vacancies posted in the submarket for active search:

$$\alpha_t = \frac{ME_{A,t} \cdot \mathbf{s}_{A,t}}{\mathbf{s}_t} = \frac{\left(\frac{\omega}{1-\omega}\right)\left(\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}\right)^{\rho}}{1 + \left(\frac{\omega}{1-\omega}\right)\left(\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}\right)^{\rho}}, \quad \rho \leq 1 \tag{41}$$

Proof. By constant returns to scale of the CES search aggregator,

$$\boldsymbol{s}_t = ME_{A,t} \cdot \boldsymbol{s}_{A,t} + ME_{P,t} \cdot \boldsymbol{s}_{P,t}$$

Then, exploiting linear homogeneity of the matching function,

$$\begin{aligned} m_t(\boldsymbol{s}_t, \upsilon_t) &= \left(\frac{ME_{A,t} \cdot \boldsymbol{s}_{A,t}}{\boldsymbol{s}_t}\right) \cdot m_t(\boldsymbol{s}_t, \upsilon_t) + \left(\frac{ME_{P,t} \cdot \boldsymbol{s}_{P,t}}{\boldsymbol{s}_t}\right) \cdot m_t(\boldsymbol{s}_t, \upsilon_t) \\ &= m_t\left(ME_{A,t} \cdot \boldsymbol{s}_{A,t}, \alpha_t \cdot \upsilon_t\right) + m_t\left(ME_{P,t} \cdot \boldsymbol{s}_{P,t}, (1-\alpha_t)\cdot \upsilon_t\right) \end{aligned}$$

where α_t is defined as in (41). □

Note, the vacancy share of active search α_t defined in Proposition 1 is analogous to a factor share from a production function. Thus, the same properties that hold for a factor share as a function of the elasticity of substitution holds for vacancy share of active search, α_t. Thus, when $\rho < 0$ and active and passive search are complements — as is the case under the estimates from the previous section — the vacancy share of active search is decreasing in the average active search of the entire non-employed, $\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}$.

The next corollary establishes that market tightness (and thus job-finding probabilities) are equal across the matching function intermediating the CES search composite and the matching functions that separately intermediate efficiency-weighted active and passive search. Thus, the share of new hires who are hired via active search is given by the vacancy share of active search, α_t.

Corollary 1 (Equal market tightness across matching functions)**.** *Consider the alternative matching structure introduced in Proposition 1. The ratio of the vacancy input to the search input, i.e. market tightness, is the same in the aggregate matching function, $m_t(s_t, \upsilon_t)$, as it is for the separate matching functions that intermediate aggregate and passive search, $m_t(ME_{A,t} \cdot \mathbf{s}_{A,t}, \alpha_t \cdot \upsilon_t)$ and $m_t(ME_{P,t} \cdot \mathbf{s}_{P,t}, (1-\alpha_t) \cdot \upsilon_t)$. Moreover, the fraction of new hires formed via the matching function intermediating active search is equal to the fraction active search share of vacancies.*

Proof. Market tightness $\theta_{A,t}$ for the matching function intermediating active search is given by the ratio of vacancies to efficiency units of search entering the matching function, i.e.

$$\theta_{A,t} \equiv \frac{ME_{A,t} \cdot s_{A,t}}{\alpha_t \cdot \upsilon_t}$$

Using the definition of the active search share of vacancies α_t, we can easily show that $\theta_{A,t} = \upsilon_t / s_t$. A similar argument establishes that market tightness $\theta_{P,t}$ for the matching function intermediating passive search is also equal to υ_t / s_t.

Then, given that market tightness θ_t in the matching function intermediating the CES search aggregate is equal to market tightness $\theta_{A,t}$ and $\theta_{P,t}$ in the notional matching functions intermediating active and passive search, we can thus establish that the vacancy filling probabilities $q_{A,t}$ and $q_{P,t}$ in the two notional submarkets is equal to the vacancy filling probability q_t for the CES search aggregate. Thus, the share of new hires matched via the active search submarket is equal to $(q_{A,t} \cdot \alpha_t \cdot \upsilon_t) / (q_t \cdot \upsilon_t)$. But given that $q_{A,t} = q_t$, the share of new hires matched via the active search submarket is equal to the active search share of new hires, α_t.

□

Given the estimates of the parameters governing the CES search aggregator and the results from Proposition 1 and Corollary 1, we have a modeling structure in place that allows us to understand how an increase in aggregate active search generates the convergence in job-finding probabilities of the active and passive non-employed shown in Figure 1.

During a recession, the fraction of non-employed engaged in active search $\check{\Gamma}_t^{ne}$ and the average active search effort $\bar{s}_{A,t}$ of the non-employed increases, as shown in the

FIGURE 3. The procyclical importance of active search

top panel of Figure 1. Given the expressions for aggregate active search in terms of $\check{\Gamma}_t^{ne}$ and $\bar{s}_{A,t}$ from equation (21), we see an increase in the ratio of aggregate active search $\boldsymbol{s}_{A,t}$ to aggregate passive search $\boldsymbol{s}_{P,t}$, as $\boldsymbol{s}_{A,t}/\boldsymbol{s}_{P,t} \equiv \check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}$. In turn, we can recognize the product of the average search of the active non-employed $\bar{s}_{A,t}$ with the fraction of active searchers among the non-employed $\check{\Gamma}_t^{ne}$ as the average search of the non-employed (inclusive of the active and passive non-employed).

Then, we can show that the time series behavior for the marginal efficiencies of active search and passive search solely depends on the average active search of the non-employed:

$$ME_{A,t} = \omega \cdot \left(\frac{\left(\omega \cdot \left(\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}\right)^{\rho} + (1-\omega)\right)^{\frac{1}{\rho}}}{\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}} \right)^{1-\rho} \tag{42}$$

and

$$ME_{P,t} = (1-\omega) \cdot \left(\omega \cdot \left(\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}\right)^{\rho} + (1-\omega)\right)^{\frac{1-\rho}{\rho}} \tag{43}$$

where make use of the expressions for $ME_{A,t}$ and $ME_{P,t}$ from (34), and constant returns to the CES aggregator.[20] Hence, the marginal efficiency of active search $ME_{A,t}$ is strictly decreasing in aggregate active search $\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}$, whereas the marginal efficiency of passive search $ME_{P,t}$ is increasing in the same quantity.

The top panel of Figure 3 shows the time series behavior for the marginal efficiency of active search, $ME_{A,t}$. The marginal efficiency of active search shows large reductions during recessions, with slow recoveries that mirror the those of unemployment. For example, the decline in the marginal efficiency of active search from about 0.35 at the beginning of 2007 to 0.12 by the end of 2009, a decrease of around two-thirds. The cyclical behavior of the marginal efficiencies of active and passive search account for the cyclical behavior of the job-finding probabilities from active and passive non-employment shown in Figure 1. The sharp recessionary decline in the average job-finding probability from active non-employment $\bar{f}_{A,t}$ reflects not only decline in the aggregate job-finding probability, but also the decline in the marginal efficiency of active search. The weaker recessionary decline in the job-finding probability from passive non-employment $f_{P,t}$ reflects the countercyclical increase in passive non-employment.

Under the model considered in Section 1, active searchers find jobs via active and passive methods. Thus, the recessionary fall in job-finding probabilities from active non-employment $f_{A,t}$ is to some degree buoyed by the increase in the marginal efficiency of passive search, $ME_{P,t}$. We can see the role of the recessionary increase

[20]Note, we omit the possibility of external time-series variation through z_t for analytic clarity.

in the marginal efficiency of passive search for job-finding outcomes through the active search share α_t. According to Proposition 1 and Corollary 1, the active search share α_t gives both the fraction of vacancies and new hires intermediated through the notional matching function for active search. Under an elasticity of substitution less than one, this share increases with the average active search of the non-employed $\check{\Gamma}_t^{ne} \cdot \bar{s}_{A,t}$.

The middle panel of Figure 3 shows large and persistent recessionary declines in the active search share of vacancies. Examining the period around the Great Recession, we see that the active search share of vacancies starts at around 0.31 prior to the Great Recession, but then falling to around 0.14 by 2009, implying a more than 50% in the notional number of vacancies intermediating active search. Thus, even though a greater share of the non-employed are searching via active methods during a recession, and the intensity of the active non-employed is greater during recessions than expansions, recessions are nonetheless periods where workers are more likely to find a job via passive search channels.

The findings here can be used here to understand empirical findings of a reduced response of unemployment to extensions of unemployment insurance during a recession. Increased UI benefits are thought to disincentive active search effort, resulting in lower job-finding probabilities and higher unemployment. However, Chodorow-Reich et al. (2018), Rothstein (2011), Farber and Valletta (2015), and Kroft and Notowidigdo (2016) find evidence suggesting that increases in unemployment insurance benefits have smaller impacts on job-finding probabilities during recessions than during normal times. The estimates here can help explain a countercyclical decreases in the distortionary effect of a UI extension: even if the disincentive effect of UI on active search is as large during a recession than during an expansion, active search is less important for job-finding during a recession due to the reduction in the marginal efficiency of active search and the lower active search share of vacancies.

4.2. **Optimal UI under a Baily-Chetty formula with a diminishing marginal efficiency of active search.** The previous section shows that the marginal efficiency of active search $ME_{A,t}$ is smaller during a recession: thus, a given increase in an individual's active search $s_{A,i,t}$ does less to increase that individual's search efficiency $s_{i,t}$ during a recession (as described in equation 35). Such a relation could be exploited by a policymaker interested in smoothing consumption of the unemployed, but who is also concerned about the disincentive effect of unemployment insurance (UI) benefits on active search. In particular, the sharp decrease in the marginal efficiency of active search during a recession suggests that the disincentive effect of UI should matter less during a recession.

A useful formula in the literature for understanding the trade-offs of consumption-smoothing versus search disincentives comes from the Bailey-Chetty formula (Baily, 1978; Chetty, 2006). The formula describes social-welfare maximizing replacement rate that is funded by taxes. Under the setting, a policy maker wishes to smooth consumption of the unemployed via UI benefits.[21] However, the policymaker cannot fully smooth consumption across unemployment and employment, as this would eliminate incentives of the unemployed to search and ultimately reduce tax revenues needed to finance UI.

The Baily-Chetty formula implicitly defines the welfare-maximizing replacement rate R:

$$\frac{d\log u}{d\log R} = \left(\frac{\mathcal{U}'(c^u)}{\mathcal{U}'(c^e)} - 1\right) \tag{44}$$

where the consumption of the employed c^e is financed by income minus taxes, the consumption of the unemployed c^u is financed by UI in the form of a replacement rate R of the previous income, $\mathcal{U}'(\cdot)$ is the marginal utility of consumption, and $d\log u/d\log R$ is the micro-elasticity of unemployment with respect to the replacement rate.

The micro-elasticity summarizes how a change in benefits effects unemployment through a reduction in active search effort. In computing the micro-elasticity, much of the existing literature adopts the simplifying assumption that workers not engaged in active search do not find jobs. This is consistent with an aggregator over active and passive search à la that given in equation (33), but with $\omega = 1$, so that $\boldsymbol{s} = \boldsymbol{s}_A$. Then, the micro-elasticity can be computed as

$$\frac{d\log u}{d\log R} = \frac{d\log u}{d\log f} \cdot \frac{d\log f}{d\log R} \tag{45}$$

The first term, $d\log u/d\log f$, is the elasticity of the unemployment rate with respect to the job-finding rate. In principle, this term can vary over time, but the term can be closely approximated by one minus the steady state unemployment rate $(1-\tilde{u})$ times negative one, given the rapid dynamics of flows in and out of unemployment (Landais et al., 2018a). The second term, $\frac{d\log f}{d\log R}$ is typically assumed constant and calibrated to estimates from the existing literature, e.g. Katz and Meyer (1990). Thus, the welfare maximizing replacement rate R^* is presumed constant.

The findings from Section 2 — along with the work of Elsby et al. (2015), Krusell et al. (2017), and Faberman et al. (2022) — question the assumption that only active searchers find jobs. Decompose the Baily-Chetty formula as follows:

$$\frac{d\log u}{d\log R} = \frac{d\log u}{d\log f} \cdot \frac{d\log f}{d\log \boldsymbol{s}} \cdot \frac{d\log \boldsymbol{s}}{d\log \boldsymbol{s}_A} \cdot \frac{d\log \boldsymbol{s}_A}{d\log R} \tag{46}$$

Compared to (45), the equation above replaces $d\log f/d\log R$ with three elasticities. Starting from the end, the fourth elasticity, $d\log \boldsymbol{s}_A/d\log R$ describes the behavioral response of active search effort to a change in unemployment benefits. The third elasticity, $d\log \boldsymbol{s}/d\log \boldsymbol{s}_A$, describes the elasticity of search efficiency to active search

[21]Note, the setting considered under the Bailey-Chetty formula is distinct from the setting considered in Section 1 of the paper. However, the estimates of the parameters of the CES aggregator introduced in Section 3 depend only on the specification of the search environment and the CES aggregator, not on the details of risk-sharing within that environment.

FIGURE 4. Optimal unemployed-employed consumption ratio Δ_t^*

effort. The second elasticity, $d\log f/d\log \boldsymbol{s}$ is the elasticity of the matching function, which typically taken as a constant.

Take the CES search aggregator proposed and estimated in Section 3 and derive

$$\frac{d\log s}{d\log s_A} = \frac{ME_A}{\boldsymbol{s}/\boldsymbol{s}_A} = \omega \cdot \left(\frac{\boldsymbol{s}_A}{\boldsymbol{s}}\right)^{\rho} \tag{47}$$

The two equations for the Baily-Chetty formula, (45) and (46), coincide only if active search is material for finding, so that $\omega = 0$ and $d\log \boldsymbol{s}/d\log \boldsymbol{s}_A = 1$. The estimates from Table (8), identified from the non-zero job-finding probabilities from passive non-employment, reject this possibility. But note, (47) also allows for a time-varying elasticity of aggregate search efficiency to aggregate active search. The elasticity is constant only if $\rho = 0$. Otherwise, the elasticity is increasing in the ratio of active-to-passive search $\boldsymbol{s}_A/\boldsymbol{s}_P$ if $\rho > 0$; and decreasing in the ratio of active-to-passive search if $\rho < 0$. Given the countercyclical ratio of aggregate active-to-passive search and our estimates of $\rho < 0$, the elasticity $d\log \boldsymbol{s}/d\log \boldsymbol{s}_A$ is thus procyclical, declining during recessions.

Thus, I compare two solutions to the Baily-Chetty formula. First, I follow the literature in abstracting from passive search, implying that (45) gives the unique welfare-maximizing replacement rate. Following Landais et al. (2018a), I use the approximation $d\log u/d\log f \approx -(1-\tilde{u})$, and then use the value $d\log f/d\log R = 0.42$, from Katz and Meyer (1990).

For the second solution, I follow the literature in assuming that all of the terms on the right-hand side of equation (46) are constant; but, consistent with the estimates of Section 3, I allow $d\log \boldsymbol{s}_t/d\log \boldsymbol{s}_{A,t}$ to vary over time. Thus, I obtain a time-varying micro-elasticity of unemployment solely through time-variation in the elasticity of aggregate search to aggregate active search. To compute the time series of the micro-elasticity, I assume the elasticity

$$\frac{d\log f_t}{d\log R_t} = \frac{d\log f}{d\log \boldsymbol{s}} \cdot \frac{d\log \boldsymbol{s}_t}{d\log \boldsymbol{s}_{A,t}} \cdot \frac{d\log \boldsymbol{s}_A}{d\log R} \tag{48}$$

has an average value of 0.42, as above. Also, I assume a matching function elasticity $d\log f/d\log \boldsymbol{s} = 0.5$. Then, using the average value of $d\log \boldsymbol{s}_t/d\log \boldsymbol{s}_{A,t}$, I back out the constant elasticity $d\log \boldsymbol{s}_A/d\log R$. Finally, I have all of the necessary terms to compute the time series $d\log f_t/d\log R_t$ from (48), and then the time series for micro-elasticity of unemployment, $d\log u_t/d\log R_t$.

Given an elasticity $d\log u_t/d\log R_t$, the optimal unemployed-employed consumption ratio $\Delta_t^* \equiv c_t^{u,*}/c_t^{e,*}$ can be computed from the Baily-Chetty formula (44) under logarithmic utility as

$$\Delta_t^* = \frac{1}{\frac{d\log u_t}{d\log R_t} + 1} \tag{49}$$

I plot the implied time series for the optimal unemployed-employed consumption ratio Δ_t^* in Figure 4 as a solid line. I also plot the optimal unemployed-employed consumption ratio $\bar{\Delta}$ from the more standard assumption that $d\log \boldsymbol{s}/d\log \boldsymbol{s}_A = 1$ (so that the micro-elasticity is constant) with the dotted line. The figure shows that optimal unemployed-employed consumption ratio Δ_t^* — derived under the estimated series for $d\log \boldsymbol{s}_t/d\log \boldsymbol{s}_{A,t}$ — is highly countercyclical. During the 2001 recession, the unemployed-employed consumption ratio increases from 0.65 to 0.80. Then, during the 2008 recession, the ratio increases from 0.72 to 0.86. These correspond to substantial increases in UI.

The countercyclical UI policy plotted in Figure 4 is due to the countercyclical elasticity of aggregate search efficiency to aggregate search effort. Recall, to compute the micro-elasticity used to produce Figure 4, we assumed that the active search disincentive was constant. Thus, the disincentive effect of UI benefits on active search are the same during a recession as during an expansion. However, the time-varying elasticity of search efficiency to active search implies that a reduction in active search effort does less to reduce search efficiency (and therefore job-finding probabilities) during a recession. Given the estimated $\rho < 1$, equation (47) describes a reduced sensitivty of aggregate search efficiency $\boldsymbol{s}_t$ to active search $\boldsymbol{s}_{A,t}$. Thus, a policy maker can do more do smooth consumption without raising unemployment.

Note, I abstract here from a separate discussion surrounding optimal UI regarding the macro-elasticity, which describes how UI affects market tightness and job-finding rates. Landais et al. (2018b) show that optimal UI incorporating a macro-elasticity prescribes less generous UI during recessions, if wages are flexible; acyclical UI (corresponding to that implied under a standard Baily-Chetty formula) if wages are rigid; and more generous UI during recessions, if wages are rigid and firms are subject to capacity constraints.[22] Likewise, Landais et al. (2018b) abstract from passive search and assume a constant micro-elasticity. Thus, an analysis combining the time-varying micro-elasticity studied here with the macro-elasticity would be fruitful, but is beyond the scope of this paper.

4.3. **An unexplored search externality.** The previous section computes optimal UI under a standard Baily-Chetty setting, where we consider how policy varies with the elasticity of aggregate search efficiency with respect to aggregate active search implied under our estimates of the CES search aggregator from Section 3. A separate, but fruitful, question is to consider the optimal allocation of a social planner within the full economic environment developed in Sections 1 and 3. The following proposition characterizes the optimal active search decision prescribed by a social planner with the objective of maximizing average utility subject to labor market frictions.

Proposition 2 (Socially optimal active search)**.** *Consider a social planner who faces a cost κ of maintaining a vacancy. Denote $\Xi_t(x,\varsigma)$ as the marginal social value in units of the consumption good of moving a non-employed worker with characteristics*

[22]Note, in comparison to the macro-elasticity, the time-varying micro-elasticity computed here is independent of the cyclicality of wages.

(x, ς) to employment at time t. The active search policy $s_{A,t}^{SP}(x, \varsigma)$ prescribed by the planner is given by $s_{A,t}^{SP}(x, \varsigma) = s_{A,t}^{SP,int}(x, \varsigma)$ if

$$\Xi_t(x, \varsigma)\Big|_{s_A^{SP}=s_A^{SP,int}} - \Xi_t(x, \varsigma)\Big|_{s_A^{SP}=0} \geq 0 \tag{50}$$

where

$$\frac{\chi}{\mu_t^{SP}} \cdot \left[s_{A,t}^{SP,int}(x, \varsigma)\right]^{\varkappa} + \kappa \cdot ME_{A,t}^{SP} \cdot \theta_t^{SP} = ME_{A,t}^{SP} \cdot f_t^{SP} \cdot \mathbb{E}_t \left\{\Lambda_{t,t+1}^{SP} \cdot \Xi_{t+1}\left(x, \varsigma_{t+1}(\varsigma)\right)\right\}$$
$$+ \frac{\partial ME_{A,t}^{SP}}{\partial \mathbf{s}_{A,t}^{SP}} \cdot f_t^{SP} \cdot \mathbb{E}_t \left\{\Lambda_{t,t+1}^{SP} \cdot cov\left(s_{A,t}^{SP}(\check{x}, \check{\varsigma}), \Xi_{t+1}(\check{x}, \varsigma_{t+1}(\check{\varsigma}))\right)\right\}, \tag{51}$$

with $s_{A,t}^{SP}(x, \varsigma) = 0$ otherwise, where the superscript "SP" on a variable ζ^{SP} denotes that the value of the variable ζ is that implied under the social planner's allocation.

Proof. See Appendix C. □

Consider interior solutions for socially optimal active search s_A^{SP}, as in equation (51), and recall the interior solutions for socially optimal active search in the decentralized economy developed in Section 1 but adapted for the CES search aggregator, i.e.

$$\frac{\chi}{\mu_t}\left[s_{A,t}^{int}(x, \varsigma)\right]^{\varkappa} = ME_{A,t} \cdot f_t \cdot \mathbb{E}_t\left\{\Lambda_{t,t+1} \cdot H_{t+1}\left(x, \varsigma_{t+1}\left(\varsigma\right)\right)\right\}. \tag{52}$$

Aggregate variables across equations (51) and (52), e.g. f_t and f_t^{SP}, will vary across the decentralized and socially optimal allocation in the presence of externalities. The comparison of $s_{A,t}^{SP}$ and $s_{A,t}$ from equations (51) and (52) elucidates considerations that enter the social planner's problem but not that of a single agent, helping to identify the sources of inefficiencies in the decentralized allocation.

The first inefficiency can be seen from the second term on the left-hand side of (52): in evaluating the marginal cost of active search, the Social Planner takes into account the social cost of maintaining a vacancy that can be matched with the searching worker. As seen from (52), the worker does not internalize this cost. This is a standard congestion externality. Under a setting without ex-ante worker heterogeneity, such an externality can be avoided if the wage received by a worker discourages excess search. The congestion externality is well known in the literature, e.g. Hosios (1990).

A second (and less familiar) inefficiency can be seen in the term incorporating the covariance of socially optimal active search s_A^{SP} and the social benefit of employment $\Xi_t(x, \varsigma)$, appearing on the second line of equation (51). As workers engage in more active search, aggregate active search $\boldsymbol{s}_A$ increases and decreases the marginal efficiency of active search at the rate $\partial ME_{A,t}/\partial \boldsymbol{s}_{A,t}$. The socially optimal level of active search takes into account that higher active search will reduce the rate at which all workers find jobs via active search.

Thus, the marginal social benefit of increasing a worker's level of active search is decreasing in (i) the degree of concavity in the marginal efficiency of active search

$ME_{A,t}$, and (*ii*) the degree of dispersion in the marginal social benefit of employment across workers who vary in the their idiosyncratic productivity x and the fixed component of the cost of active search ς. The first component can be seen in the term $\partial ME^{SP}_{A,t}/\partial \boldsymbol{s}^{SP}_{A,t}$, which is negative when active and passive search enter the matching function as imperfect substitutes. The second can be seen from the covariance term. Given that (51) indicates that the socially optimal active search of a worker with characteristics (x, ς) is increasing in the marginal social value of moving a worker from non-employment to employment, the covariance term is increasing in the variance in the marginal social value of moving a worker from non-employment to employment across the population. Thus, when there is more variation in the marginal value of employment across workers, the social planner prescribes less search to low marginal value workers as to not impede the rate at which workers with a high marginal value of employment find a job.

Note, this second source of inefficiency — associated with a crowding-out of active search – is absent from the rest of the literature, as it is typically assumed that active and passive search are perfect complements. Under the standard assumption, $\partial ME^{SP}_{A,t}/\partial \boldsymbol{s}^{SP}_{A,t} = 0$, and so such an inefficiency does not appear.

5. Conclusion

To be written.

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Appendix A. Micro evidence for the returns to search

Section 2 uses aggregate data to show that the job-finding rate from active non-employment relative to the job-finding rate from passive non-employment is declining in active search. This section goes to the micro-level data used to construct the aggregate series applied in the previous section. I develop a surprising finding from a regression equation motivated by the theory: the job-finding probability of active searchers is decreasing in their active search effort. Once we allow that the returns to individual search are decreasing in active search, however, we recover the expected increasing relationship of job-finding probabilities and active search effort. Such a finding is consistent with a diminishing marginal efficiency of active search, as developed and estimated from Section 3

Recall equation (19) from Section 1 for the job-finding rate of an individual i under a Cobb-Douglass matching function:

$$f_{i,t} = \Big(\omega \cdot s^A_{i,t} + (1-\omega)\Big) \cdot \varphi_t \theta_t^{1-\eta}$$

where $f_{i,t}$ is the job-finding rate, $s^A_{i,t}$ is active search effort, ω and $1-\omega$ represent the marginal efficiencies of active and passive search, $\varphi_t \theta_t^{1-\eta}$ is the aggregate job-finding rate, where the aggregate job-finding rate itself is a function of matching efficiency φ_t, market tightness θ_t, and matching elasticity η.

Taking logs, we obtain

$$\log f_{i,t} = \log \Big(\omega \cdot s^A_{i,t} + (1-\omega)\Big) + \log \varphi_t + (1-\eta) \cdot \log \theta_t \tag{A.1}$$

Of the three terms in (A.1), the second and third are the only to contain time-varying aggregate variables. Notably, the only individual specific variable is $s^A_{i,t}$, which is contained in the first term. Thus, to a logarithmic transformation, the job-finding probability is additively separable in individual and aggregate characteristics.

Even though active search $s^A_{i,t}$ can be zero, the first logarithm of equation (A.1) is well-defined, as it will be evaluated at the marginal efficiency of passive search $1-\omega$. In practice, however, this parameter is unobserved. Thus, I propose a regression equation in levels to isolate the role of individual active search effort for job-finding outcomes for individual i in region r at date t:

$$\mathbb{I}\left\{FindJob_{i,r,t+1}\right\} = \gamma_0 + \gamma_1 \cdot s^A_{i,t} + x'_{i,t}\beta + \gamma_t + \gamma_r + \varepsilon_{i,r,t} \tag{A.2}$$

where $\mathbb{I}\left\{FindJob_{i,r,t+1}\right\}$ is an indicator variable taking the value one if an individual i in region r in active or passive non-employment at date t is employed at date $t+1$, $x_{i,t}$ is a vector of individual level characteristics, γ_t is a time fixed-effect, and γ_r is a region fixed-effect. Although the (A.2) does not represent a true structural equation from a model, it respects the separability of individual and aggregate variables implied under the structural equation (A.1).

The first column of Table A.1 contains the coefficient estimate of γ_1, where number of search methods is taken as a measure of active search effort. The estimated coefficient is positive and precisely estimated, but small in magnitude, suggesting only a small role for active search in increasing the probability of finding a job.

Table A.1. Active search at the individual level

	Dependent variable: Indicator for moving to employment in subsequent period		
	(1)	(2)	(3)
$s^A_{i,t}$ × relative quantity of active search in aggr.	0.006*** (0.0003)	−0.002*** (0.0004)	0.028*** (0.0048)
$s^A_{i,t}$ × relative quantity of active search in aggr.	—	—	−0.003*** (0.0005)
$\mathbb{I}\{s^A_{i,t} = 0\}$	—	−0.040*** (0.0013)	−0.202*** (0.0125)
$\mathbb{I}\{s^A_{i,t} = 0\}$ × relative quantity of active search in aggr.	—	—	0.016*** (0.0012)
N	865,079	865,079	865,079
Time fixed effects?	Yes	Yes	Yes
Region fixed effects?	Yes	Yes	Yes

Note: Sample of active and passive searchers, 1996-2019. Includes controls for education, gender, race, marital status, a quartic for age, and fixed-effects for time and region.

Note, although the sample includes both workers in active and passive non-employment, regression equation (A.2) does not distinguish between the two populations, except when the variable s^A_t takes on the value zero. Thus, I estimate a second equation that includes an indicator variable for workers not engaged in active search:

$$\mathbb{I}\left\{FindJob_{i,r,t+1}\right\} = \gamma_0 + \gamma_1 \cdot s^A_{i,t} + \gamma_2 \cdot \mathbb{I}\left\{s^A_{i,t} = 0\right\} + x'_{i,t}\beta + \gamma_t + \gamma_r + \varepsilon_{i,r,t} \quad \text{(A.2a)}$$

The second column of Table A.1 offers estimates of γ_1 and γ_2 from the regression specification (A.2a). The coefficient on the indicator variable for the passive non-employed is negative and large in magnitude, indicating a penalty from purely passive search larger than that predicted in the previous specification. More surprisingly, however, the coefficient for active search effort is estimated to be negative: greater search effort is associated with lower job-finding probabilities.[23]

Note, the negative estimated coefficient on active search from the second column of Table A.1 is subject to multiple interpretations. For example, it may indicate that workers who inherently have a more difficult time finding a job compensate by searching harder.[24] Alternatively, the negative coefficient estimate for "active search"

[23]The negative relationship of search effort and job-finding probabilities is preserved when search effort is introduced in logs, or if a quadratic term in search effort is added to the regression.

[24]If we observed workers with two distinct spells of non-employment, we could control for such individual-specific attributes with fixed effects. The structure of the CPS makes this difficult (if

could also be consistent with a diminishing marginal efficiency of active search. As documented in Section 2, active search effort rises during recessions. If this rise is accompanied by a reduction in the marginal efficiency of active search, the negative coefficient from the second column could be driven by the co-occurrence of low job-finding probabilities, high active search effort, and a low marginal efficiency of active search.

Consider an alternative regression specification,

$$\begin{aligned}\mathbb{I}\left\{FindJob_{i,r,t+1}\right\} = \gamma_0 + \gamma_1 \cdot s^A_{i,t} + \gamma_{1,x} \cdot \left(\frac{\bar{s}^A_t \cdot ne_{A,t}}{s_t}\right) \cdot s^A_{i,t} \\ + \gamma_2 \cdot \mathbb{I}\left\{s^A_{i,t} = 0\right\} + \gamma_{2,x} \cdot \left(\frac{\bar{s}^A_t \cdot ne_{A,t}}{s_t}\right) \cdot \mathbb{I}\left\{s^A_{i,t} = 0\right\} \\ + x'_{i,t}\beta + \gamma_t + \gamma_r + \varepsilon_{i,r,t}\end{aligned} \tag{A.2b}$$

with

$$s_t = \bar{s}^A_t \cdot ne_{A,t} + ne_{P,t}, \tag{A.3}$$

where s_t is total search efficiency, $\bar{s}^A_t$ is average search effort per active non-employed, $ne_{A,t}$ is the total number of active non-employed, and $ne_{P,t}$ is the total number of passive non-employed.[25] Thus, the term $\left(ne_{A,t} \cdot \bar{s}^A_t\right)/s_t$ provides a measure of the relative quantity of active search in the aggregate. While the independent effect of the relative quantity of active search is subsumed by the time-fixed effects, the interaction term allows for the relative quantity of active search to reduce or increase the efficiency of an individual's active search effort for finding a job, as captured by the estimated coefficient $\gamma_{1,x}$.

Estimates for equation (A.2b) are given in the third column of Table A.1. While the estimated coefficient on the indicator for purely passive searchers remains negative, the estimated coefficient on active search effort becomes positive once again. Notably, the point estimate of the coefficient is nearly five times higher than in the first column of Table A.1. The estimated coefficients on the interaction terms involving the relative quantities of active search can be interpreted as evidence for a "crowding-out" effect of active search: when the relative measure of active search goes up by a standard deviation, the returns to active search go down by about a tenth, and the penalty to purely passive search reduces by about the same fraction.

Note, however, that the regression equation (A.2b) for the coefficient estimates from the third column of Table (A.1) breaks the logarithmic separability of individual and aggregate variables implied under equation (19). This suggests the need for an alternative mapping of active search effort into individual job-finding probabilities where the return to active search depends on the relative contribution of aggregate active search to aggregate search efficiency.

not impossible) to do; and moreover, one might be concerned that such individuals would offer a non-representative sample of the non-employed.

[25] The expression for $\left(ne_{A,t} \cdot \bar{s}^A_t\right)/s_t$ in equation (A.3) is what one would obtain under the linear search aggregator (**??**) with $\omega = 0.5$.

Such a mapping is provided under the unrestricted CES aggregator of Section 3, where the effect of individual active search effort on the probability of finding a job is scaled by the composition of aggregate search, e.g. equation (36).

Appendix B. The household problem

The representative family has a unit measure of unemployed and employed family members, representative of the population at large. The family pools income before choosing consumption to maximize present discounted value of the family, taking the active search decision of non-employed workers as given.

Let $\Omega_t(x,\varsigma)$ be the present discounted value at time t of the average household worker with characteristics (x,ς). Denote

$$\Omega_t \equiv \int_{X\times\mathcal{C}} \Omega_t(x,\varsigma) d\Gamma_x(x) d\Gamma_\varsigma(\varsigma)$$

as the present discounted value of the household over workers in the household. Then, the problem of the household at time t is

$$\Omega_t = \max_{c_t(\varsigma,x)} \int_{X\times\mathcal{C}} \Big[\mathcal{U}\left(c_t(x,\varsigma,1), s_{A,t}(x,\varsigma,1)\right)\cdot d\Upsilon_t(x,\varsigma) \\ +\mathcal{U}\left(c_t(x,\varsigma,0), s_{A,t}(x,\varsigma,0)\right)\cdot\left(d\Gamma_x(x)d\Gamma_\varsigma(\varsigma) - d\Upsilon_t(x,\varsigma)\right)\Big] + \beta\mathbb{E}_t\Omega_{t+1} \tag{B.4}$$

subject to the budget constraint,

$$\int_{X\times\mathcal{C}} \Big[c_t(x,\varsigma,1)\cdot d\Upsilon_t(x,\varsigma) + c_t(x,\varsigma,0)\cdot\left(d\Gamma_x(x)d\Gamma_\varsigma(\varsigma) - d\Upsilon_t(x,\varsigma)\right)\Big] = \int w_t(x,\varsigma)\cdot d\Upsilon_t(x,\varsigma) \tag{B.5}$$

and the law of motion for Υ_t, as defined in equations (10) and (11) of the main text.

Given separability of utility from consumption and leisure, the first-order condition for optimal consumption implies uniform consumption within the household:

$$\mathcal{U}_c\left(c_t(x,\varsigma,1)\right) = \mathcal{U}_c\left(c_t(s,\varsigma,0\right) = \mu_t \quad \forall\ (x,\varsigma)\in X\times\mathcal{C} \tag{B.6}$$

where μ_t is the multiplier on the budget constraint.

To calculate the marginal surplus of employment to the household at time t of workers with characteristics (x,ς), differentiate (B.4) with respect to $d\Upsilon_t(x,\varsigma)$:

$$\frac{\partial\Omega_t}{\partial[d\Upsilon_t(x,\varsigma)]} = \mathcal{U}\left(c_t(x,\varsigma,1), s_{A,t}(x,\varsigma,1)\right) - \mathcal{U}\left(c_t(x,\varsigma,0), s_{A,t}(x,\varsigma,0)\right) \\ + w_t(x,\varsigma)\cdot\mu_t \\ + \beta\mathbb{E}_t\Bigg\{\left(1 - f_t(x,\varsigma) - \delta\right)\left[(1-\lambda)\frac{\partial\Omega_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]} + \lambda\int_{\mathcal{C}}\frac{\partial\Omega_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma')]}d\Gamma_\varsigma(\varsigma')\right]\Bigg\} \tag{B.7}$$

by the law of motion for $d\Upsilon$, equations (10) and (11).

Denote the consumption-equivalent household surplus of moving a worker with characteristics (x, ς) at time t from non-employment to employment as

$$\frac{\partial \tilde{\Omega}_t}{\partial \Upsilon_t(x,\varsigma)} \equiv \frac{1}{\mu_t} \cdot \frac{\partial \Omega_t}{\partial \Upsilon_t(x,\varsigma)}$$

Then, from (B.7), obtain a recursive equation for $\partial \tilde{\Omega}_t / \partial [d\Upsilon_t(x,\varsigma)]$

$$\begin{aligned} \frac{\partial \tilde{\Omega}_t}{\partial \Upsilon_t(x,\varsigma)} &= w_t(x,\varsigma) \\ &- \frac{1}{\mu_t}\left(\psi - \varsigma \cdot \mathbb{I}\left\{s_{A,t}(x,\varsigma) > 0\right\} - \chi \cdot \frac{\left[s_{A,t}(x,\varsigma)\right]^{1+\kappa}}{1+\kappa}\right) \\ &+ \mathbb{E}_t\Bigg\{\Lambda_{t,t+1} \cdot (1 - f_t(x,\varsigma) - \delta) \\ &\cdot \left[(1-\lambda)\frac{\partial \tilde{\Omega}_{t+1}}{\partial [d\Upsilon_{t+1}(x,\varsigma)]} + \lambda \int_{\mathcal{C}} \frac{\partial \tilde{\Omega}_{t+1}}{\partial [d\Upsilon_{t+1}(x,\varsigma')]} d\Gamma_\varsigma(\varsigma')\right]\Bigg\}. \end{aligned} \tag{B.8}$$

Appendix C. Social Planner problem

Let $P_t(\varsigma, x)$ be the present discounted value at time t of the average worker with idiosyncratic productivity x and cost of active search ς. Denote

$$P_t \equiv \int_{X\times\mathcal{C}} P_t(x,\varsigma) d\Gamma_x(x) d\Gamma_\varsigma(\varsigma)$$

as the present discounted value of the household over workers in the economy. Then, the problem of the social planner is

$$\begin{aligned} P_t = \max_{c_t(x,\varsigma,e), s_{A,t}(x,\varsigma,e), \theta_t} \int_{\mathcal{C}} \Big[&\mathcal{U}\left(c_t(x,\varsigma,1), s_{A,t}(x,\varsigma,1)\right) \cdot d\Upsilon_t(x,\varsigma) \\ &+ \mathcal{U}\left(c_t(x,\varsigma,0), s_{A,t}(x,\varsigma,0)\right) \cdot \left(d\Gamma_x(x) d\Gamma_\varsigma(\varsigma) - d\Upsilon_t(x,\varsigma)\right)\Big] + \beta \mathbb{E}_t P_{t+1} \end{aligned} \tag{C.9}$$

subject to the resource constraint,

$$\begin{aligned} &\int_{X\times\mathcal{C}} \Big[c_t(x,\varsigma,1) \cdot d\Upsilon_t(x,\varsigma) + c_t(x,\varsigma,0) \cdot \left(d\Gamma_x(x) d\Gamma_\varsigma(\varsigma) - d\Upsilon_t(x,\varsigma)\right)\Big] \\ &= \int_{X\times\mathcal{C}} y_t \cdot x \cdot d\Upsilon_t(x,\varsigma) - \int_{X\times\mathcal{C}} \kappa \cdot \theta_t \Big(d\Gamma_x(x) d\Gamma_\varsigma(\varsigma) - d\Upsilon_t(x,\varsigma)\Big) s_t(x,\varsigma) \end{aligned} \tag{C.10}$$

and the law of motion for $d\Upsilon$, equations (10) and (11).

The equilibrium allocation in the Social Planner's problem can be described by (i) the marginal social value of employment for a worker with a fixed cost of search ς, $\partial P / \partial [d\Upsilon(\varsigma)]$, (ii) a policy function for active search, $s_A(\varsigma)$, and (iii) a value of market tightness, θ, that solve (C.9).

First, solve for consumption:

$$\mathcal{U}_c\left(c_t(x,\varsigma,1)\right) = \mathcal{U}_c\left(c_t(x,\varsigma,0)\right) = \mu_t \quad \forall (x,\varsigma) \in X \times \mathcal{C} \tag{C.11}$$

given separable utility in consumption and leisure, where μ_t is the multiplier on the resource constraint.

Take derivatives with respect to $d\Upsilon_t(x,\varsigma)$:

$$\begin{aligned}\frac{\partial P_t}{\partial[d\Upsilon_t(x,\varsigma)]} =& \mathcal{U}\left(c_t(x,\varsigma,1), s_{A,t}(x,\varsigma,1)\right) - \mathcal{U}\left(c_t(x,\varsigma,0), s_{A,t}(x,\varsigma,0)\right) \\ &+ \mu_t\left(y_t \cdot x + \kappa \cdot \theta_t \cdot s_t(x,\varsigma)\right) \\ &+ \beta\left(1 - f_t(x,\varsigma) - \delta\right) \times \mathbb{E}_t\Bigg\{(1-\lambda)\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}\frac{\partial[d\Upsilon_{t+1}(x,\varsigma)]}{\partial[d\Upsilon_t(x,\varsigma)]} \\ &+ \lambda\int_{X\times\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(\check{x},\check{\varsigma})]}\frac{\partial[d\Upsilon_{t+1}(\check{x},\check{\varsigma})]}{\partial[d\Upsilon_t(x,\varsigma)]}d\Gamma_x(\check{x})d\Gamma_\varsigma(\check{\varsigma})\Bigg\}\end{aligned} \tag{C.12}$$

Solve for optimal market tightness, θ_t:

$$\begin{aligned}\mu_t \cdot \kappa \cdot \boldsymbol{s}_t =& \beta\mathbb{E}_t\Bigg\{\int_{X\times\mathcal{C}}\Bigg((1-\lambda)\cdot\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}\frac{\partial[d\Upsilon_{t+1}(x,\varsigma)]}{\partial\theta_t} \\ &+ \lambda\int_{\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\check{\varsigma})]}\frac{\partial[d\Upsilon_{t+1}(x,\check{\varsigma})]}{\partial\theta_t}d\Gamma_\varsigma(\check{\varsigma})\Bigg)\varsigma\Bigg\} \\ =& \beta\cdot\boldsymbol{s}_t\cdot f'\left(\theta_t\right)\cdot\beta\cdot\mathbb{E}\Bigg\{\int_{X\times\mathcal{C}}\Bigg((1-\lambda)\cdot\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]} + \lambda\int_{\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\check{\varsigma})]}d\Gamma_\varsigma(\check{\varsigma})\Bigg)\varsigma\Bigg\}\end{aligned} \tag{C.13}$$

Then recall,

$$f_t'(\theta_t) = q_t(\theta_t)\left(1 - \epsilon_t\right)$$

where ϵ_t is the elasticity of the matching function with respect to aggregate search efficiency. Multiply by θ_t, and then suppressing the dependence of f_t and ϵ_t on θ_t in the notation, we obtain

$$\mu_t\kappa\theta_t = f_t\cdot(1-\epsilon_t)\cdot\beta\cdot\mathbb{E}_t\Bigg\{\int_{X\times\mathcal{C}}\Bigg((1-\lambda)\cdot\frac{\partial P_{t+1}}{\partial[d\Upsilon_t(x,\varsigma)]} + \lambda\int_{\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\check{\varsigma})]}d\check{\varsigma}\Bigg)\Bigg\} \tag{C.14}$$

Finally, take first-order conditions for $s_{At}(x,\varsigma)$:

$$\begin{aligned}&(\chi\cdot[s_{A,t}(x,\varsigma)]^{\varkappa}+\mu_t\cdot\kappa\cdot ME_{A,t}\cdot\theta_t) \qquad (C.15)\\ &=\beta\cdot ME_{A,t}\cdot f_t\cdot\beta\mathbb{E}_t\left\{(1-\lambda)\frac{\partial P_{t+1}}{\partial[d\Upsilon_t(x,\varsigma)]}+\lambda\int_{\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_t(x,\varsigma')]}d\Gamma_\varsigma(\varsigma')\right\}\\ &+\beta\cdot f_t\cdot\int_{X\times\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(\check{x},\check{\varsigma})]}\left[\frac{\partial[d\Upsilon_{t+1}(\check{x},\check{\varsigma})]}{\partial ME_{A,t}}\cdot\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\cdot s_{A,t}(\check{x},\check{\varsigma})\right.\\ &\left.+\frac{\partial[d\Upsilon_{t+1}(\check{x},\check{\varsigma})]}{\partial ME_{P,t}}\cdot\frac{\partial ME_{P,t}}{\partial\boldsymbol{s}_{A,t}}\right]d\Gamma_x(\check{x})d\Gamma_\varsigma(\check{\varsigma})\end{aligned}$$

By homogeneity of degree zero, we can write

$$\begin{aligned}\frac{\partial ME_{P,t}}{\partial\boldsymbol{s}_{A,t}}&=-\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\int_{X\times\mathcal{C}}s_{A,t}(x,\varsigma)d\Gamma_x(x)d\Gamma_\varsigma(\varsigma)\\ &=-\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\boldsymbol{s}_{A,t}\end{aligned}$$

and thus

$$\begin{aligned}&f_t\cdot\beta\mathbb{E}_t\left\{\int_{\mathcal{C}}\frac{\partial P_{t+1}}{\partial\Upsilon_{t+1}(\check{x},\check{\varsigma})}\left[\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}s_{A,t}(\check{x},\check{\varsigma})+\frac{\partial ME_{P,t}}{\partial\boldsymbol{s}_{A,t}}\right]d\Gamma_{x(\check{x})}d\Gamma_\varsigma(\check{\varsigma})\right\} \qquad (C.16)\\ &=\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\cdot f_t\cdot\beta\mathbb{E}_t\left\{\int_{X\times\mathcal{C}}\left(s_A(\check{x},\check{\varsigma})\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(\check{x},\check{\varsigma}]}\right)d\Gamma_x(\check{x})d\Gamma_\varsigma(\check{\varsigma})\right.\\ &\left.-\boldsymbol{s}_{A,t}\int_{\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(\check{x},\check{\varsigma})]}d\Gamma_x(\check{x})d\Gamma_\varsigma(\check{\varsigma})\right\}\\ &=\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\cdot f_t\cdot\beta\cdot\mathrm{cov}\left(s_{A,t}(x,\varsigma),\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}\right)\end{aligned}$$

Substituting (C.16) into (C.15), we obtain

$$\begin{aligned}&\chi\cdot[s_{A,t}(x,\varsigma)]^{\varkappa}+\mu_t\cdot\kappa\cdot ME_{A,t}\cdot\theta_t \qquad (C.17)\\ &=ME_{A,t}\cdot f_t\cdot\beta\mathbb{E}_t\left\{(1-\lambda)\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}+\lambda\int_{\mathcal{C}}\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma'))]}d\Gamma_\varsigma(\varsigma')\right\}\\ &+\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\cdot f_t\cdot\beta\cdot\mathbb{E}_t\left\{\mathrm{cov}\left(s_{A,t}(x,\varsigma),\frac{\partial P_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}\right)\right\}\end{aligned}$$

Finally, denote $\tilde{P}_t\equiv P_t/\mu_t$ as the social value of workers in the economy in units of the consumption good, and $\Xi(x,\varsigma)\equiv\partial\tilde{P}_t(x,\varsigma)/\partial\left[d\Upsilon_t(x,\varsigma)\right]$ to be the marginal social value in consumption units of moving a worker with characteristics (x,ς) from unemployment to employment. Then,

$$\begin{aligned}&\frac{\chi}{\mu_t}\cdot[s_{A,t}(x,\varsigma)]^{\varkappa}+\kappa\cdot ME_{A,t}\cdot\theta_t \qquad \text{(C.18)}\\&\quad=ME_{A,t}\cdot f_t\cdot\mathbb{E}_t\left\{\Lambda_{t,t+1}\left[(1-\lambda)\frac{\partial\tilde{P}_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}+\lambda\int\limits_{\mathcal{C}}\frac{\partial\tilde{P}_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma'))]}d\Gamma_\varsigma(\varsigma')\right]\right\}\\&\quad+\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\cdot f_t\cdot\mathbb{E}_t\left\{\Lambda_{t,t+1}\cdot\operatorname{cov}\left(s_{A,t}(x,\varsigma),\frac{\partial\tilde{P}_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}\right)\right\}\end{aligned}$$

Thus, the active search policy $s_{A,t}(x,\varsigma)$ prescribed by the planner is given by $s_{A,t}(x,\varsigma)=s^{int}_{A,t}(x,\varsigma)$ if

$$\frac{\partial P_t}{\partial d[\Upsilon_t(x,\varsigma)]}\bigg|_{s_A=s^{int}_A(\varsigma)}-\frac{\partial P_t}{\partial d[\Upsilon_t(x,\varsigma))]}\bigg|_{s_A=0}\geq 0 \qquad \text{(C.19)}$$

where

$$\begin{aligned}&\frac{\chi}{\mu_t}\cdot\left[s^{int}_{A,t}(x,\varsigma)\right]^{\varkappa}+\kappa\cdot ME_{A,t}\cdot\theta_t \qquad \text{(C.20)}\\&\quad=ME_{A,t}\cdot f_t\cdot\mathbb{E}_t\left\{\Lambda_{t,t+1}\left[(1-\lambda)\frac{\partial\tilde{P}_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}+\lambda\int\limits_{\mathcal{C}}\frac{\partial\tilde{P}_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma'))]}d\Gamma_\varsigma(\varsigma')\right]\right\}\\&\quad+\frac{\partial ME_{A,t}}{\partial\boldsymbol{s}_{A,t}}\cdot f_t\cdot\mathbb{E}_t\left\{\Lambda_{t,t+1}\cdot\operatorname{cov}\left(s^{int}_{A,t}(x,\varsigma),\frac{\partial\tilde{P}_{t+1}}{\partial[d\Upsilon_{t+1}(x,\varsigma)]}\right)\right\}\end{aligned}$$

with $s_{A,t}(x,\varsigma)=0$ otherwise.