Temporary and Permanent Layoffs over the Business Cycle: Evidence, Theory, and an Application to the Covid-19 Crisis

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Preliminary, comments welcome

Abstract

Motivated by the unusual increase in temporary unemployment during the recent recession, this paper develops a quantitative model of unemployment dynamics that distinguishes between temporary and permanent layoffs. We calibrate the model to capture labor market dynamics over the period from 1979 to 2019. We then adapt the full quantitative model to study the effects of the extraordinary increase in temporary layoffs induced by the pandemic. We also use the model to evaluate how the Paycheck Protection Program may have worked to facilitate the return of workers to employment from temporary layoff. We find that, without PPP, unemployment would have been persistently higher: Firms would have recalled far fewer workers from temporary layoff, and more workers on temporary layoff would have drifted into more persistent unemployment.
1 Introduction

A salient and well-documented feature of the recent pandemic recession was the massive inflow of workers into temporary layoff. As shown in Figure 1, roughly fifteen percent of workers in employment moved to temporary layoff from March to April 2020, the onset of the recession. The increase in temporary layoffs was an aggregate phenomenon that spared no sector of the U.S. economy’s workforce, as can be seen from Figure 2.

An immediate concern of policymakers and economists was that the sharp increase in temporary layoffs might translate into large and persistent increases in unemployment if workers on temporary layoff were to lose connection to their previous employers. Thus, Congress passed the Payroll Protection Program (PPP), which comprised the largest single component of the federal government fiscal response to the pandemic. The program delivered forgivable loans to firms to encourage the recall of workers from temporary layoff, with the broader goal of keeping temporary layoff “temporary.” While the labor market has improved over the last year and a half, it is not possible to tell from the raw time series alone how successful the PPP program was. Doing so, ideally, requires a structural model.

We accordingly develop a general equilibrium model of unemployment fluctuations with two types of unemployment: temporary-layoff unemployment, consisting of unemployed workers waiting to be recalled to their previous employer; and jobless unemployment, consisting of unemployed workers who are permanently separated from their previous employer and thus are searching for new jobs.\(^1\) The rates at which workers move from temporary-layoff and jobless unemployment to employment depend on the recall and hiring decisions of the firm. The flows of workers from employment to temporary-layoff and jobless unemployment, in turn, depend on the decisions of firms to place workers on temporary layoff or shut down\(^2\). Although wages are set according to multi-period contracts, firms may cut wages temporarily to avoid shutdown. If a firm shuts down, all of the firm’s workers in temporary-layoff unemployment move to jobless unemployment. The resulting model offers the necessary ingredients for capturing employment dynamics during the crisis, as well evaluating the employment effect of PPP.

Though it is the recent recession that provides the main motivation for

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\(^1\)We adopt this terminology from Hall and Kudlyak (2020).

\(^2\)We use the term “shutdown” to refer to job destruction in general. As we discuss when presenting the model, what we call a firm may also be a plant or shift within a plant. Within our framework, shutdown and job destruction are equivalent.
our model, temporary-layoff unemployment has shaped the cyclical dynamics of unemployment to various degrees across all of the postwar recessions, as recently noted by Fujita and Moscarini (2017). During a recession, temporary layoffs increase and recalls fall, generating an increase in temporary-layoff unemployment, and also total unemployment.

We document a separate, indirect channel by which temporary separations increase total unemployment: the rate at which temporary-layoff unemployment resolves to jobless unemployment is itself highly countercyclical. Hence, a recessionary spike in temporary layoffs increases jobless unemployment. We derive accumulation equations that allow us to estimate the contribution of temporary separations to jobless unemployment using matched monthly data from the CPS. We show that the increase in jobless unemployment from a recessionary spike in temporary separations is substantial, in some cases similar in magnitude to the contemporaneous increase in temporary-layoff unemployment.

We calibrate our model to match the dynamics of temporary unemployment, permanent unemployment, hiring, recalls, and separations using CPS data from 1979 to 2019. Our model does well at matching the data. We establish from our model that the division of unemployment into temporary and permanent allows firms to adjust more flexibly to aggregate shocks, thereby decreasing the persistence of total unemployment.

We then adapt the model to study the labor market at the onset of the Covid-19 pandemic. We show that the model can capture well the dynamics of total unemployment as well as the breakdown between temporary-layoff and jobless unemployment. In addition to capturing the evolution of the stocks, the model also explains well the flows between different types of labor market status over the recession. We then apply our model to evaluating the effectiveness of PPP. To do so, we use the quantitative model to analyze a counterfactual scenario where PPP is removed.

The analysis shows that PPP was successful in fulfilling its immediate objective of preserving ties between firms and workers on temporary layoff. From May to October 2020, firms recalled twice as many workers as they would have in the absence of PPP; and half as many workers in temporary layoff lost their previous position permanently as would have without PPP. Accordingly, we estimate substantial employment effects of PPP over the same period, with monthly employment gains from PPP averaging 2.14 percent. We estimate the employment effects of PPP to be highly persistent, with employment still around one percentage point higher in May 2021 than it would have been absent PPP.

After reviewing the related literature, in Section 2 we present our model
with three labor market states and five endogenous flows. In Section 3, we calibrate the model to CPS labor market data from 1979 to 2019 and examine its predictions for the dynamics of permanent and temporary unemployment, as well as the flows underlying them. In Section 4, we apply the model to the Covid-19 recession. We first adapt the model to certain specific features of the pandemic crisis and estimate a series of “fundamental” shocks to capture the economic consequences of the crisis. We then rely on the structure of the model to study the response of the labor market to the pandemic fundamentals, as well as to PPP. We finally evaluate the contribution of PPP to limiting permanent employment losses by studying how the labor market would have responded in a counterfactual economy without PPP. Concluding remarks are in Section 5.

**Related literature.** Our paper builds on several diverse strands of literature:

On the empirical side, a large recent literature documents the employment landscape in the months following the onset of the pandemic. A common theme is the emphasis on the importance of how transitions in and out of temporary unemployment will shape subsequent labor market dynamics. Contributions include Barrero, Bloom, and Davis (2020), Chodorow-Reich and Coglianese (2021), Cajner et al. (2020), Coibion et al. (2020), Gallant, Kroft, Lange, and Notowidigdo (2020), Hall and Kudlyak (2020), and Şahin and Tasci (2020). We complement this empirical literature with a more structural approach that ties the labor market stocks and flows to primitive model parameters and is suitable for counterfactual policy evaluation.

In addition, our work is complementary to a reduced-form empirical literature that uses firm-level data to estimate the aggregate employment effect of PPP, e.g. Hubbard and Strain (2020), Chetty et al. (2020) and Autor et al. (2020). Our paper offers an additional contribution by using our structural model to assess how the immediate disbursement of PPP money generated persistently lower unemployment. As we show from our model, the longer-term recovery of employment depends on the composition of workers across temporary unemployment and permanent employment. Thus, by explicitly incorporating dynamic dependencies and general equilibrium effects, our structural model allows us to assess the medium- to long-run impact of PPP on employment.

On the modeling side, our approach fits into the literature on DSGE models of unemployment with wage rigidity, e.g. Shimer (2005), Hall (2005), Gertler and Trigari (2009) and Christiano, Eichenbaum, and Trabandt (2016). We
differ in two important ways. First, following Fujita and Ramey (2012), we allow for endogenous separations from employment. Second, in the spirit of Fujita and Moscarini (2017) we allow for recall hiring as well as hiring of new workers. We differ in some important details, however. Fujita and Moscarini consider recalls across all workers in unemployment and document that recalls of such workers are countercyclical. In contrast, motivated by the critical role of temporary layoffs in the most recent recession, we instead focus on recalls of workers from temporary layoff, and we document that recalls of these workers are highly procyclical.

Also highly relevant is the work by Gregory, Menzio, and Wiczer (2020), which is the first attempt to our knowledge to quantify the role of temporary unemployment. These authors emphasize the role of heterogeneity across industries in worker employment stability. In addition to differing significantly in details, we develop a framework that can capture labor market dynamics both pre and post recession. We also offer a formal analysis of PPP and show how it helped shape the employment recovery.

Our paper is also related to Birinci et al. (2020), who develop a general model by which to compare the welfare benefits of UI extensions versus payroll subsidies during a pandemic. Our papers differ along several important dimensions: Birinci et al. (2020) study a rich model that accommodates rich heterogeneity and multiple policy instruments, but abstract from business cycle dynamics and endogenous recalls from temporary-layoff unemployment. As such, the authors do not use their model to study the full dynamics of unemployment across the multiple waves of the Covid-19 pandemic, and thus do not offer a full quantitative evaluation of PPP.

Finally, as in the robust recent literature on the interaction between disease contagion and economic activity, our aim is to study the unusual dynamics of how the recent recession has played out, (see e.g., Alvarez, Argente, and Lippi (2020), Eichenbaum, Rebelo, and Trabandt (2020), Farboodi, Jarosch, and Shimer (2020), Kaplan, Moll, and Violante (2020), Birinci et al. (2021), and the references therein). For reasons of tractability, we do not incorporate an epidemiological model. We instead represent the pandemic as a set of exogenous shocks affecting the economy, thus ignoring the feedback from economy behavior to the spread of the virus, an approach similar to Guerrieri, Lorenzoni, Straub, and Werning (2020), for example.
2 Empirics

We begin by offering new evidence for the importance of temporary-layoff unemployment in determining aggregate labor market dynamics. Our primary data source is the monthly Current Population Survey (CPS), from 1978-2021. We use longitudinally linked monthly surveys to construct data on gross worker flows across labor market states as in Blanchard and Diamond (1990), Shimer (2012), and Elsby, Hobijn, and Sahin (2016). Given the historically unprecedented spike in temporary layoffs beginning in 2020, we exclude 2020 and 2021 from our sample when documenting the historical behavior of temporary layoffs. We return to the most recent recession at the end of our analysis.

We begin by presenting summary statistics for stocks, including total unemployment $u$, jobless unemployment $u_{JL}$, and temporary-layoff unemployment $u_{TL}$. Table 1 provides the average values of these stocks, as well as measures of their cyclical properties. As can be seen from the table, both jobless and temporary layoff unemployment are countercyclical and highly volatile. However, temporary-layoff unemployment is shown on average to account for approximately one-eighth of total unemployment. One might conclude from this observation that temporary layoffs play a only small role in shaping overall unemployment dynamics. The rest of our discussion establishes that this is not so.

The stocks of these three labor market states are determined by the probabilities of moving across the various stocks. Hence, although the stock of workers in temporary-layoff unemployment may be small, this does not indicate that flows to and from temporary-layoff unemployment are also small. Thus, we estimate a Markov transition matrix between employment, jobless unemployment, and temporary-layoff unemployment. In doing so, we abstract from labor market inactivity, as is common in the literature on unemployment fluctuations. To generate the desired three-state Markov transition matrix, we first estimate separate a time series of transition probabilities across four states: employment, jobless unemployment, temporary-layoff unemployment, and inactivity. After correcting for time-aggregation bias as in Shimer (2012) and Elsby, Hobijn, and Sahin (2015), we then “condition out” transitions to inactivity so that transition probabilities from a given labor force state to employment, jobless unemployment, and temporary-layoff unemployment sum to one. Similar to the two-state method proposed by Shimer (2012), the resulting transition probabilities imply a series of “stochastic steady states” for jobless

\[^3\text{CPS respondents are asked if they have any expectation of returning to work in the next six months. Respondents answering in the affirmative are categorized as temporary-layoffs.}\]
and temporary-layoff unemployment that align well with those observed in the data.\textsuperscript{4} We then compute a monthly Markov transition matrix by averaging across the entire time series of transition probabilities.

The resulting Markov transition matrix is given in Table 2. We immediately see that separations to temporary-layoff unemployment account for roughly one-third of all separations. Thus, temporary layoff is indeed important in accounting for separations from employment and the dynamics of total unemployment.\textsuperscript{5} Accordingly, the stock of workers in temporary-layoff unemployment is relatively small because it is a relatively transient state. The transition matrix shows that this is due to two sources: First, workers on temporary-layoff return to employment at an extremely high rate. Second, conditional on not returning to employment, workers in temporary-layoff unemployment have a relatively high probability of exiting to jobless unemployment. Unlike temporary-layoff unemployment, jobless unemployment is a relatively persistent state: workers move to employment from jobless unemployment at a substantially lower rate than from temporary-layoff unemployment.\textsuperscript{6}

Note, we measure a spell of temporary-layoff unemployment as having resolved to jobless unemployment if a CPS survey respondent indicates that they no longer expect to be recalled to work. Accordingly, we interpret movements from temporary-layoff unemployment to jobless unemployment as reflecting a loss of the worker’s recall option. A related paper by Fujita and Moscarini (2017) studies recall and temporary layoff from the Survey of Income and Program Participation (SIPP), which measures whether a worker expects to be recalled only at the point of separation. Hence, their study necessarily abstracts from the “loss of recall” that we document in Table 2. However, Fujita and Moscarini document that workers who exit employment due to temporary layoff have declining recall probabilities the longer they remain unemployed.

\textsuperscript{4}Fujita and Moscarini (2017) use the Shimer (2012) two-state method with the CPS to estimate separate transition probabilities between employment and temporary-layoff unemployment; and between employment and jobless unemployment. Such an application of Shimer’s methodology restricts that the probability of moving from temporary to permanent unemployment to be zero. As we will show, our estimate for the probability of moving from temporary to permanent unemployment is non-zero and countercyclical, suggesting the importance of such flows.

\textsuperscript{5}Temporary layoff is one of the six reasons that the CPS uses to classify unemployment.

\textsuperscript{6}Note that workers can move from jobless to temporary-layoff unemployment, with probability 0.027. Given the small fraction of workers in temporary unemployment at a given time (see Table 1), we regard such observations as driven by measurement error and not statistically significant. This accords with our model, where the probability of moving from permanent to temporary unemployment is zero.
Our estimates rationalize such negative duration dependence as coming from loss of recall.

Finally, we establish the importance of temporary layoffs for explaining the cyclical volatility of total unemployment. We seasonally adjust the transition probabilities underlying the Markov transition matrix in Table 2, take quarterly averages, and then apply an HP-filter with smoothing parameter 1600. Table 3 reports the standard deviations of the resulting series relative to HP-filtered GDP, as well as correlations with HP-filtered GDP.

The table illustrates a direct effect and indirect effect of temporary separations on unemployment. During a recession, temporary layoffs increase, and exits from temporary-layoff unemployment to employment fall. This allows an increase in temporary-layoff unemployment, thus increasing total unemployment. We refer to this as the “direct effect,” and its role in boosting the countercyclicality of total unemployment has been documented elsewhere in the literature: see, for example, Shimer (2012). This direct effect is not the only way that temporary separations contribute to the countercyclicality of total unemployment, however. During a recession, the rate at which workers move from temporary-layoff unemployment to jobless unemployment increases. This generates what we term the “indirect effect:” a recessionary increase in temporary separations generates greater jobless unemployment, also contributing to the cyclicality of total unemployment.

How does this indirect effect of temporary layoff, whereby heightened “loss-of-recall” shifts the composition of unemployment from temporary-layoff to jobless unemployment, contribute to the countercyclicality of total unemployment over the business cycle? To answer this question, we derive a series of recursive accumulation equations that allow us to estimate a time series for the fraction of workers in jobless unemployment whose most recent exit from employment is due to temporary layoff. The necessary inputs for producing the desired time series are the various stocks and transition probabilities estimated for the previous tables. A full derivation of the accumulation equations is provided in Appendix A2.

Figure 1 provides a decomposition of unemployment across three large recessionary periods in our data: the 1980’s recessions, the Great Recession, and the 2020 recession. At each point in time, we express total unemployment as the sum of three components: i) the stock of workers in temporary-layoff unemployment, ii) workers in jobless unemployment whose most recent employment-exit is due to temporary layoff, and iii) workers in jobless unemployment whose most recent employment exit is not due to temporary layoff. The existing literature focuses only on this first component as the contribution of temporary layoff to the dynamics of total unemployment. The innovation
of our approach is to document the existence of the second component, and estimate its quantitative contribution to total unemployment.

The figure shows that the importance of temporary layoffs varies across recessions. During the 1980’s recessions, temporary layoffs account for 36.1\% of the total increase in unemployment. The expansion of temporary-layoff unemployment contributes towards 25.1\% of the increase in total unemployment, whereas the contribution from an expansion in jobless unemployment due to loss-of-recall — the indirect effect — accounts for the remaining 11.0\%. During the Great Recession, temporary-layoff plays a smaller role in shaping overall unemployment dynamics, accounting for 17.2\% of the total increase in total unemployment. Here, the size of the direct and indirect effects are roughly similar, with the former accounting for 8.7\% and the latter contributing 8.5\% towards the total increase. It is worth emphasizing, though, that the smaller fraction of workers in temporary unemployment during the Great Recession is not due to a decrease in inflows to temporary unemployment, but rather to higher outflows to employment: That is, a trend increase in recall hiring is what dampened the rise in temporary unemployment. Compare Figures A.1 and A.2 of the Appendix.

In sum, for both the 1980s and 2007-2008 recessions, the indirect effect of temporary separations on total unemployment was larger than the direct expansion in temporary-layoff unemployment. The indirect effect has thus far gone unquantified in the empirical literature; thus we believe that the importance of temporary layoffs has been understated. As will become clear, for the Covid-19 recession, the dynamics of the indirect effect are of particular significance.

The final panel of Figure 1 shows the contribution of temporary layoffs to increase in unemployment during the Covid-19 recession. Here, of course, temporary-layoff unemployment accounts for nearly all of the initial increase in unemployment at the onset of the Covid-19 recession. Moreover, nearly all of the persistent increase in jobless-unemployment comes from workers whose employment exit was due to temporary layoff, i.e. the indirect effect is important. Note also that the Covid-19 recession was different from other large recessions in a number of ways. First, the recession is the only one where the stock of workers in temporary-layoff unemployment is at any point greater than in jobless unemployment. Second, it is only recession in our data where the contribution of separations to total unemployment through an expansion in temporary unemployment exceeds the contribution through jobless unemployment. As we will see from the model, some of the unusual feature of the recession are due to fundamental economic forces that hit the labor market at the onset of the pandemic, such as the onslaught of temporary layoffs. Other
features are due to the unprecedented fiscal response through PPP, which prevented temporary separations from resolving into jobless unemployment.

We develop the model in the next section of the paper. Then, we calibrate the model to match features of the pre-2020 data. Finally, we adopt the model to study the role of temporary layoffs in the Covid-19 labor market.

3 Model

Our starting point is the Diamond, Mortensen, and Pissarides search and matching framework, modified to allow for wage rigidity in the form of staggered multiperiod contracting, as in Gertler and Trigari (GT). To this framework, we add two main features: First we allow for endogenous employment separations, which we refer to as layoffs. Second, we make the distinction between temporary and permanent layoffs. As a result, firms can expand their labor force through both recalls from temporary-layoff unemployment and new hires from jobless unemployment. Moreover, workers in temporary-layoff unemployment can transition to jobless unemployment either exogenously through time or because their job is destroyed. Figure 3 illustrates the stocks and flows within the model.

Next we describe the labor market of the model and then turn to a description of the full general equilibrium.

3.1 Search, matching and recalls

There are a continuum of firms and a continuum of workers, each of measure unity. For each firm $i$ operating in the current period, let $n_i$ and $u_{TL}$ be beginning of period employment and temporary-layoff unemployment and let $v_i$ be vacancies the firm posts during the period. The corresponding aggregate values are $\bar{n} = \int n_i \, di$, $\bar{u}_{TL} = \int u_{TL} \, di$ and $\bar{v} = \int v_i \, di$. Let $\bar{u}_{JL}$ be the total number workers in “jobless” unemployment (i.e. unemployed workers not currently attached to a firm). Then, given a total population of unity:

$$1 = \bar{u}_{JL} + \bar{u}_{TL} + \bar{n}. \quad (1)$$

During the period, each firm hires a continuum of workers and operates a constant returns to scale technology. Given the homothetic technology, firms’ decisions, including hiring, layoffs and default choices, are independent of it’s scale, as measured by it’s current stock of beginning of period employment $n$. Although we continue to refer to production units as a “firm,” note that
within our model there will be no practical distinction between a firm and a
plant (or perhaps between a plant and an assembly line).

Employment grows in two ways: hiring from permanent unemployment and
recalls from temporary unemployment. Analogously, employment declines in
two ways: endogenous permanent layoffs and endogenous temporary layoffs.
For simplicity, we abstract from exogenous permanent separations.

In the model, overhead costs give rise to endogenous separations. A firm
enters the period with a stock of workers \( n \) plus knowledge of the aggregate
shocks. The firm and its workers then receive two types of overhead cost
shocks. The first is a worker specific cost shock \( \vartheta \). As will become clear in the
next subsection, the firm puts on temporary layoff workers with a shock above
an endogenously determined threshold \( \vartheta^* \). It chooses to put the worker on
temporary as opposed to permanent layoff for two reasons: First the worker’s
job is not destroyed since the shock is worker-specific. Second, we assume
the shock is transitory, meaning that at some point it may be profitable to
reemploy that worker.

The firm then receives a firm-specific cost shock \( \gamma \), which has a common
effect on costs across all its workers. The firm must pay the overhead costs to
operate. Accordingly, as we describe in the next section, for values of this shock
above an endogenously determined threshold \( \gamma^* \), the firm exits, destroying all
the jobs. The firm’s workers then go into permanent unemployment. Because
within our model there is no practical distinction between a firm and a plant,
exit may refer either to bankruptcy or a plant/branch shutdown. Conditional
on exit, the workers then go on permanent layoff, which moves them into
permanent unemployment.

Both \( \gamma \) and \( \vartheta \) are i.i.d. and lognormally distributed over the range \([0, \infty)\),
where \( G(\gamma) \) and \( F(\vartheta) \) denote the respective cumulative distribution functions.
Then by defintion, the probability a worker does not go temporary layoff \( F \)
and the probability the firm does not exit \( G \) are given by, respectively.

\[
F = F(\vartheta^*). \tag{2}
\]
\[
G = G(\gamma^*). \tag{3}
\]

Given \( F \) and \( G \), we can describe the labor market flows. Let: \( x \) be the hiring
rate from permanent unemployment and \( x_r \) the hiring rate from temporary
unemployment. Further we use “bars” to denote the averages of \( x \) and \( x_r \).
Then the evolution of aggregate employment is given by

\[
\bar{n}' = (1 + \bar{x} + \bar{x}_r) \overline{G F} \bar{n}, \tag{4}
\]
where $\overline{GF}$ is the probability a worker avoids both permanent and temporary unemployment during the period, averaged across firms. It follows that $\overline{GF}\overline{n}$ is total employment used in production in the current period.

We next turn to flows in and out temporary unemployment. Workers in temporary unemployment may either (i) stay; (ii) return to employment; or (iii) move to permanent unemployment. For simplicity, we assume that the only way a worker in temporary unemployment can return to employment is via recall: The worker does not search for a job at another firm while on temporary unemployment.\(^7\) The worker can also move to permanent unemployment in one of two ways: First they separate from temporary unemployment at the exogenous rate $1 - \rho_r$. Second, if the firm to which they are attached exits, they move to permanent unemployment. Finally, they enter temporary unemployment in one of two ways. First, as just discussed, the endogenous fraction $1 - F$ of workers at surviving firms are put on temporary layoff. Second, as we discuss later, if there is a lockdown due to the pandemic, a fraction of the workforce entering the period moves to temporary unemployment.

Let $p_r$ be the (endogenous) recall rate. Then we can express the evolution of temporary unemployment as

$$\overline{u}'_{TL} = \rho_r (1 - \overline{p}_r) \overline{GF} \overline{u}_{TL} + \overline{GF} (1 - \overline{F})\overline{n},$$

(5)

where the average recall rate out of temporary unemployment $\overline{p}_r$ is linked to firms’ average hiring rate out of temporary unemployment $\overline{x}_r$, as follows:

$$\overline{p}_r = \frac{\overline{x}_r \cdot \overline{F} \overline{n}}{\overline{u}_T}.$$

(6)

We show in the next section how each firm chooses its hiring rate $x_r$ and implicitly its recall rate $p_r$.

We now complete the description of the labor market flows. The matching function for permanent unemployed and vacancies is given by

$$\bar{m} = \sigma_m (\bar{u}_P)^\sigma (\bar{v})^{1-\sigma}.$$

(7)

The job filling and finding rates, in turn, are given by

$$q = \frac{\bar{m}}{\bar{v}},$$

(8)

\(^7\)We have experimented with allowing workers in temporary unemployment to search for outside employment. However, taking into account the high rate at which workers on temporary layoff return to their previous employer (as documented by Fujita and Moscarini 2017), we have found that including this additional margin has no apparent change on the quantitative implications of our model.
Finally, the hiring rate from permanent unemployment is given by

\[ \bar{x} = \frac{q \bar{v}}{\bar{gF}n} = \frac{p \bar{u}}{\bar{gF}n}. \]  

(10)

3.2 Firms

3.2.1 Hiring and temporary layoff for non-exiting firms

Here we consider the hiring and temporary layoff decisions of a firm operating in the current period. In the next section we consider the bankruptcy/exit decision. As before we let \( n \) denote the firm’s stock of workers at the beginning of the period, \( 1 - \mathcal{F}(\vartheta^*) \) the fraction the firm placed on temporary layoff, and \( \mathcal{F}(\vartheta^*)n \) the effective labor force. Recall that \( \vartheta^* \) is the threshold value of \( \vartheta \), where for realizations of \( \vartheta^* \), the worker goes on temporary layoff.\(^8\) It follows that by choosing \( \vartheta^* \), the firm is choosing the fraction of workers that go on temporary layoff.

Technology and Constraints  Each firm produces output \( y \) using a Cobb-Douglas production function, using labor not on temporary layoff \( \mathcal{F}(\vartheta^*)n \) and capital \( k \) as inputs. Let \( \bar{z} \) be total factor productivity and \( \xi_k \) and \( \xi_n \) the exogenously given rates of capital and labor utilization. Then output is given by

\[ y = \bar{z}(\xi_k)^{\alpha}(\xi_n\mathcal{F}(\vartheta^*)n)^{1-\alpha} \]

(11)

\[ = \bar{z}k^{\alpha}(\mathcal{F}(\vartheta^*)n)^{1-\alpha}, \]

where \( z \) is effective productivity and where, for simplicity, capital is perfectly mobile across firms. We suppose that \( \bar{z} \) obeys the following first order process

\[ \log \bar{z} = \rho z \log \bar{z} + \epsilon' \]

(12)

where \( \epsilon' \) is \( i.i.d \) with mean zero and standard deviation \( \sigma_{\bar{z}} \). For the time being we take \( \xi_k \) and \( \xi_n \) as fixed. When we turn to analyzing the pandemic recession, we capture social distancing effects on productivity as reductions in the the effective rate of input utilization, following Kaplan, Moll and Violante (2020).\(^9\)

\(^8\)To ease notation we abstract from the dependence of the thresholds \( \gamma^* \) and \( \theta^* \) on \( (w, s) \).

\(^9\)The social distancing behavior could come from either formal restrictions or voluntary aversion to the virus.
For a non-exiting firm, the evolution of the firm’s employment depends on its’ hiring rate $x$, its’ recall rate $x_r$ and its’ stock of available workers, $F(\vartheta^*)n$, as follows

$$n' = (1 + x + x_r)F(\vartheta^*)n,$$

(13)

The stock of the firm’s workers in temporary layoff unemployment is given by

$$u'_TL = \rho_r u_{TL} - \rho_r x_r F(\vartheta^*)n + (1 - F(\vartheta^*))n$$

(14)

This stock varies in inversely with recall hiring $x_r F(\vartheta^*)n$ and positive with the fraction of the firm’s workers newly added to temporary unemployment, $1 - F(\vartheta^*)$. We add that the firm’s recall hiring cannot exceed the stock of its’ workers on temporary layoff.

$$x_r F(\vartheta^*)n \leq u_{TL}$$

(15)

In choosing $x$, $x_r$ and $\vartheta^*$, the firms faces both overhead costs and hiring costs. As described the previous subsection, overhead costs depend on a worker-specific cost shock $\vartheta$ realized in the beginning of the period and a firm-specific cost shock $\gamma$ realized later on. Given $\vartheta^*$ is the firm’s threshold value of $\vartheta$, we suppose that overhead costs $\varsigma(\gamma, \vartheta^*)n$ are proportionate to the firms beginning of period employment $n$, as follows.

$$\varsigma(\gamma, \vartheta^*)n = \left(\varsigma_\gamma \gamma + \varsigma_\vartheta \int_{\vartheta^*}^{\vartheta} \vartheta dF(\vartheta)\right)n,$$

(16)

where $\varsigma_\gamma$ and $\varsigma_\vartheta$ are parameters, and where $\int_{\vartheta^*}^{\vartheta} \vartheta dF(\vartheta)$ is the sum of worker–specific costs shocks over active employees. According to equation (16), overhead costs are increasing in both $\gamma$ and $\vartheta^*$. Finally, as we have noted, for the firm to be operating, $\gamma$, cannot exceed endogenously determined threshold, $\gamma^*$, which we characterize in the next section.

We suppose that hiring and recall costs depend on the respective hiring rates and are both proportionate to the effective labor force, measured by the stock of workers not on temporary layoff $x$:

$$\iota(x)Fn = \left[\chi x + \frac{\kappa}{2} (x - \bar{x})^2\right]Fn$$

$$\iota_r(x_r)Fn = \left[\chi x_r + \frac{\kappa_r}{2} (x_r - \bar{x}_r)^2\right]Fn$$

(17)

where $\bar{x}$ and $\bar{x}_r$ are the steady state values of the hiring rates and $\varsigma(\vartheta^*, \gamma)n$ is given by equation (16).
We assume that hiring costs out of each type of unemployment are the sum of a linear and a quadratic term. We allow the respective coefficients on the quadratic term, $\kappa$ and $\kappa_r$, to differ. This permits us to flexibly estimate elasticities of hiring with respect to firm value separately for hiring from unemployment versus recalls. As we will show, we capture the idea that hiring out of temporary unemployment is relatively less costly by estimating a higher elasticity for recall hiring than for new worker hiring.

Hiring and separations also depend on wages. Let $w$ be the base contract wage the firm faces in period $t$. We assume that wage bargaining is on a staggered basis and elaborate later on how is $w$ determined. We also allow for temporary paycuts to reduce the likelihood of a firm exit. For example, if due to a large negative shock to profitability the firm is not able to meet the base wage payment and remain solvent, then a temporary paycut is possible. Accordingly, the firm faces a wage schedule $\omega(w, \gamma, s)$, where the wage depends on the base wage, the firm-specific idiosyncratic cost shock, and the state of the economy. We defer a derivation of the wage schedule to the next section. In the meantime, note that the firm cannot cut the wage below workers’ reservation wage. If it cannot meet the reservation wage, it exits (as we describe in the next section.) In addition, we assume all workers receive the same wage: i.e. the firm cannot condition a worker’s wage on his or her idiosyncratic cost shock.

**Timing of Events** Overall, during each period, the firm and its workers face three shocks: the effective productivity shock $z$, the worker-specific cost shock $\vartheta$, and the firm-specific productivity shock $\gamma$. Before continuing to the firm’s decision problem, it is useful to clarify the intra-period timing, given as follows:

1. The aggregate shock is realized.
2. Bargaining over base wages and state-contingent provisions for temporary paycuts may take place. Otherwise the firm takes as given the wage schedule $\omega(w, \gamma, s)$ from the previous period.
3. The employee-specific cost shock is realized and the firm adds to temporary layoff unemployment the fraction $1 - F(\vartheta^*)$ of its workers.
4. The firm-specific cost shock $\gamma$ is realized. With probability $1 - G(\gamma^*)$ the firm exits, implying that both is current workers and its workers on temporary layoff move into permanent unemployment. With probability $G(\gamma^*)$, the firm continues. though. It rents capital, produces and
pay wages. Temporary paycuts are possible if the realization of \( \gamma \) is sufficiently low.

5. The firm recalls workers from temporary layoff unemployment and hires new workers. The jobless unemployment search. Those on temporary layoff unemployment lose their recall option with probability \( 1 - \rho_r \).

**Decision Problem** To solve the firm’s decision problem we work backwards, beginning in the middle of the period after the realization of \( \gamma \). At this point the firm has decided its’ layoff policy \( \vartheta^* \). As we noted earlier, because both production and costs are homogenous of degree one in labor, we can express the decision problem in terms of the firm maximizing value per worker. Let \( J(w, \gamma, \bar{u}_{TL}, \vartheta^*, s) \) be firm value per worker; i.e., firm value divided by and let \( \bar{J}(w', \bar{u}_{TL}', s') \) be the expected firm value per worker in the subsequent period, prior to the realization of \( \gamma' \) and the choice of a layoff policy \( \vartheta'^* \). Next, let \( \bar{k} \) and \( \bar{u}_{TL} \) be capital and temporary layoff unemployment, each relative to the effective labor force:

\[
\bar{k} = \frac{k}{F(\vartheta^*)n}, \quad (18)
\]

\[
\bar{u}_{TL} = \frac{u_{TL}}{F(\vartheta^*)n}, \quad (19)
\]

and, finally, let \( r \) be the rental rate on capital. Then, given \( \vartheta^* \), the problem of a non-exiting firm (one with a realization of \( \gamma \) below \( \gamma^* \)) is to choose \( \bar{k}, x, x_r, \) and \( \bar{u}_{TL} \) to solve

\[
J(w, \gamma, \bar{u}_{TL}, s) = \max_{\bar{k}, x, x_r, \bar{u}_{TL}} \left\{ zF(\vartheta^*)\bar{k}^\alpha - \omega(w, \gamma, s)F(\vartheta^*) - r\bar{k}F(\vartheta^*) \right. \\
- (\iota(x) + \iota_r(x_r))F(\vartheta^*) - \zeta(\vartheta^*, \gamma) \\
+ F(\vartheta^*) (1 + x + x_r) \mathbb{E} \left\{ \Lambda(s, s') \bar{J}(w', \gamma', \bar{u}_{TL}', s') \right\},
\]

subject to equations (14), (15), (16), and (17). The top term on the right is revenue minus labor and capital compensation, all per worker. The middle term is adjustment and overhead costs per worker. The bottom term is the expected discounted value of per worker value next period.

Finally, we solve for the optimal value of \( \vartheta^* \) prior to the realization of \( \gamma \) by solving

\[
\bar{J}(w, \bar{u}_{TL}, s) = \max_{\vartheta^*} \int_{\gamma^*}^{\gamma} J(w, \gamma, u_{TL}, s)dG(\gamma), \quad (21)
\]
equation where (20) defines \( J(w, \gamma, u_T, s) \). In choosing \( \vartheta^* \) the firm trades off the benefit of having fewer workers on temporary layoff versus the increase in overhead costs. We derive the exit threshold \( \gamma^* \) in the next section.

Before proceeding we make an important technical simplification. As we show in the appendix, the constraint that recalls cannot exceed temporary unemployment does not bind under a first order approximation of the estimated model. Intuitively, the quadratic hiring costs dampen recall hiring sufficiently to keep the constraint from binding. Hence, to a first order, the problem where the firm ignores the constraints on recall hiring generates the same allocations as the problem described in (20). Thus, we can restrict attention to the simpler case where equation (15) does not bind. Accordingly, the first order conditions below are derived for the case where the recall constraint is never binding.\(^\text{10}\)

The first order conditions for the hiring rates \( x \) and \( x_r \), are given by

\[
\chi + \kappa (x - \bar{x}) = \mathbb{E} \left\{ \Lambda (s, s') \bar{J} (w', s') | w, s \right\}, \quad (22)
\]

\[
\chi + \kappa_r (x_r - \bar{x}_r) = \mathbb{E} \left\{ \Lambda (s, s') \bar{J} (w', s') | w, s \right\}, \quad (23)
\]

Equations (22) and (23) imply that both hiring from jobless unemployment and recalls from temporary layoff unemployment depend positively on discounted firm value. The volatilities of \( x \) and \( x_r \) depend on the respective adjustment cost parameters, \( \kappa \) and \( \kappa_r \). One can show that to a first order approximation, the elasticity of \( x \) with respect to discounted firm value is \( \chi/\kappa \bar{x} \), while \( x_r \) it is \( \chi/\kappa_r \bar{x}_r \). As discussed later we estimate each elasticity. We find that the recall elasticity exceeds the hiring elasticity, consistent with the notion that it is less costly for firms to adjust employment via recalls than hire from jobless unemployment.

Next, the first order condition for the threshold for temporary layoffs \( \vartheta^* \) is given by

\[
\bar{J}(w, s) + \varsigma_{\gamma} \int_{\gamma}^{\gamma^*} \gamma dG(\gamma) + \varsigma_{\vartheta} G(\vartheta^*) \int_{\vartheta}^{\vartheta^*} \vartheta dF(\vartheta) = \varsigma_{\vartheta} \vartheta^* F(\vartheta^*) G(\gamma^*), \quad (24)
\]

The left side of (24) is the marginal benefits of increasing \( \vartheta^* \), i.e. the marginal benefit of keep more workers employed and off temporary layoff. The right side is the marginal cost, i.e. the marginal increase in overhead costs from keeping more workers employed.

\(^{10}\)Effectively, we are ignoring precautionary behavior by the firm to avoid the recall constraint on the grounds that to a first order the likelihood of hitting the constraint is remote. Note, if (15) does not bind, it is straightforward to show that (14) does not bind as well. Given our focus on cases in which (15) does not bind, we can suppress \( u_T \) in the firm value function.
For capital renting $\tilde{k}$, the first order is standard

$$\alpha z\tilde{k}^{\alpha - 1} = r, \quad (25)$$

Finally, using the hiring conditions and the capital renting condition, we get the following expression for value per worker in an operating firm after temporary layoffs:

$$J(w, \gamma, s) = a - \omega(w, \gamma, s) - \frac{\zeta(\vartheta^*, \gamma)}{\mathcal{F}(\vartheta^*)}$$

$$+ \frac{\kappa}{2} (x^2 - \tilde{x}^2) + \frac{\kappa_r}{2} (x^2_r - \tilde{x}^2_r)$$

$$+ \mathbb{E} \{ \Lambda(s, s') \tilde{J}(w', \gamma', s') | w, \tilde{u}_T, s \},$$

with

$$a = (1 - \alpha)z\tilde{k}^\alpha.$$ 

Firm value per worker includes saving on adjustment costs from having a worker already in the firm.

### 3.2.2 Firm Exit and Near Exit

As we discussed, workers move into permanent unemployment when the firm (or plant or shift) at which they are employed exits. Exit occurs when the firm is insolvent. In turn, near bankruptcy is a situation where a temporary wage cut can allow the firm to escape insolvency. We assume that if the worker takes a temporary paycut, the worker’s pay reverts to the base wage in subsequent periods. Given the form the wage schedule takes, firms and workers negotiate multiperiod wage contracts on a staggered basis, as we discuss in section 3.4.

In particular, we assume a wage schedule consists of three elements: first, a base wage $w$ that the worker receives in normal times; second, a “temporary pay cut” wage $w^†(w, \gamma, s)$ the worker receives if the firm cannot afford the base wage (due to a high realization of the firm-specific idiosyncratic shock $\gamma$); and third, a reservation wage $w(w, s)$, which is the lowest wage the worker will accept. Accordingly, we can express the wage schedule $\omega(w, \gamma, s)$ as:

$$\omega(w, \gamma, s) = \begin{cases} 
  w & \text{if } \gamma \leq \gamma^†(w, s) \\
  w^†(w, \gamma, s) & \text{if } \gamma^†(w, s) < \gamma < \gamma^*(w, s) \\
  w(w, s) & \text{if } \gamma = \gamma^*(w, s)
\end{cases} \quad (27)$$

with $w > w^†(w, \gamma, s) \geq w(w, s)$. 

18
The threshold for exit is the realization of the idiosyncratic shock \( \gamma^* \) at which the firm value per worker is zero when the current wage is reduced to workers’ reservation value \( w(w, s) \). Accordingly, \( \gamma^* \) solves\(^{11}\)

\[
J(w, \gamma^*(w, s), s) = 0. \tag{28}
\]

Given how \( \gamma^* \) is determined, it follows that for realizations of \( \gamma \) above \( \gamma^* \), firm value per worker is negative, leading the firm to exit. In the next section we describe how the reservation wage \( w(w, s) \) is determined.

We turn to the determination of \( w^\dagger(w, \gamma, s) \), the current wage when the realization of \( \gamma \) lies between the paycut threshold \( \gamma^\dagger \) and the bankruptcy cutoff \( \gamma^* \). With \( \gamma \in (\gamma^\dagger, \gamma^*) \), overhead costs are low enough for the firm to avoid bankruptcy: But it needs to engineer a temporary wage cut to stay solvent. We suppose for simplicity, that when a temporary paycut is necessary, it is the minimum needed to keep the firm solvent. As a result the paycut keeps firm value per worker at zero. We can then trace out the wage schedule conditional on \( \gamma \in (\gamma^\dagger, \gamma^*) \).

We start with the determination of the temporary paycut threshold \( \gamma^\dagger(w, s) \). This threshold is the value of \( \gamma \) at which firm value is zero, given the current wage is the base contract wage \( w \). This condition is given by

\[
J\left(w, \gamma^\dagger(w, s), s\right) = 0. \tag{29}
\]

Next, for any value of \( \gamma \in (\gamma^\dagger, \gamma^*) \), we can determine the “paycut wage” \( w^\dagger(w, \gamma, s) \), using the requirement that the pay cut keeps value per worker at zero. Accordingly, \( w^\dagger(w, \gamma, s) \) satisfies

\[
J(w, \gamma, s) = 0. \tag{30}
\]

In section 3.4 we describe how base wages are determined by staggered multiperiod wage bargains. In bargaining over base wages, firms and workers take account of the paycut policy, as well as the reservation wage for workers.

### 3.3 Worker Value Functions and the Reservation Wage

Let \( V(w, \gamma, s) \) and \( U_{TL}(w, s) \) be the values of employment and temporary-layoff unemployment for a worker at a non-exiting firm, and let \( U_{JL}(s) \) be the value of jobless unemployment.

\(^{11}\)Note that, given the definition of \( J(w, \gamma, s) \) in (26) and that of the wage schedule \( \omega(w, \gamma, s) \) in (27), this implies evaluating \( J \) in (28) at the reservation wage \( w(w, s) \) to solve for \( \gamma^*(w, s) \).
The value of work at a non-exiting firm is given by
\[ V(w, \gamma, s) = \omega(w, \gamma, s) + \mathbb{E}\{\Lambda(s, s') \tilde{V}(w', s')|w, s\}, \]  
(31)
where \( \omega(w, \gamma, s) \) is the wage schedule defined in the previous section and \( \tilde{V}(w, s) \) is the expectation of the value of work prior to the realization of both \( \vartheta \) and \( \gamma \), given by
\[ \tilde{V}(w, s) = \mathcal{F}(\vartheta^*) \left[ \int_{s'}^{s} V(w, \gamma, s) dG(\gamma) + (1 - G(\gamma^*) \right) U_{JL}(s) \right] 
+ (1 - \mathcal{F}(\vartheta^*)) \tilde{U}_{TL}(w, s), \]  
(32)
The first term on the right is the product of the probability the worker is not put on temporary layoff, \( \mathcal{F}(\vartheta^*) \), and the expected gain from being in this situation. The latter is the sum of the expected gain from working - which depends on the probability the firm survives - and the probability the firm exits, \( 1 - G(\gamma^* (w, s)) \), times the value of unemployment. The second term is the probability the worker is put on temporary layoff times the expected value of being in this state \( \tilde{U}_{TL}(w', s') \), where the expectation is taken prior to the realizations of \( \vartheta \) and \( \gamma \).

Let \( b \) be unemployment insurance per period. Then we can express the value of temporary-layoff unemployment as
\[ U_{TL}(w, s) = b + \mathbb{E}\{\Lambda(s, s') \mathcal{P}_r \tilde{V}(w', s') \]  
\[ + (1 - \mathcal{P}_r) \rho_r \tilde{U}_{TL}(w', s') \]  
\[ + (1 - \mathcal{P}_r) (1 - \rho_r) U_{JL}(s') |w, s\} \]  
(33)
with
\[ \tilde{U}_{TL}(w, s) = G(\gamma^*) U_{TL}(w, s) + (1 - G(\gamma^*)) U_{JL}(s). \]  
(34)
Then the value of temporary-layoff unemployment is the sum of \( b \) and the expected discounted value of the laid-off worker’s future state. The latter is the sum of the expected discounted value of being recalled (the top right term in 33), the expected discounted value of staying in temporary layoff unemployment (the middle term), and the expected discounted value of moving to permanent unemployment (the bottom term). In turn, \( \tilde{U}_{TL}(w, s) \) is convex combination of \( U_{TL}(w, s) \) and \( U_{JL}(s) \), where the weights are the probability the firm survives \( G(\gamma^* (w, s)) \) and the probability it exits \( 1 - G(\gamma^* (w, s)) \).

Next let \( b \) be unemployment insurance per period. Then we can express the value of jobless unemployment \( U_{JL}(s) \) as
\[ U_{JL}(s) = b + \mathbb{E}\{\Lambda(s, s') [p \tilde{V}_{x}(s') + (1 - p) U_{JL}(s')] |s\}, \]  
(35)
where \( p \) is the job-finding probability adjusted for search intensity and where \( \bar{V}_x(s) \) is the expected value of being a new hire, given by\(^{12}\)

\[
\bar{V}_x(s') = \int_w \bar{V}(w', s') \frac{x(w, s) + x_r(w, s)}{\bar{x} + \bar{x}_r} dW(w, s).
\]

We can then express the surplus from employment and the expected surplus from employment for a non-exiting firm as follows:

\[
H(w, \gamma, s) \equiv V(w, \gamma, s) - U_{JL}(s),
\]

Finally, we can characterize the determination of the reservation wage. At the reservation wage \( w(w, s) \), the worker’s surplus from employment is zero:

\[
H(w, \gamma, s) = 0.
\]

That is, we find a value for \( \omega(w, \gamma, s) = w(w, s) \) that satisfies equation (38).

3.4 Wage bargaining

We assume following GT that a firms and its workers bargain over wages on a multiperiod, staggered basis. Let \( 1 - \lambda \) be the probability the parties negotiate a new contract in a given period. This realization of this random draw is independent across time and across firms. When able, the parties bargain over a base wage, taking into account both the temporary pay cut rule described in section ?? and the possibility of exit. The base wage then remains in place until the firm and its workers are able again to renegotiate.

As noted earlier, bargaining takes place after the realization of the aggregate shock but prior to the idiosyncratic costs shocks. With probability \( 1 - \lambda \), the parties negotiate a new base wage \( w'' \). With probability \( \lambda \) the parties are unable to negotiate. In this case, the contract wage from the previous period, \( w \) along with the wage schedule \( \omega(w, \gamma, s) \) remains intact. Accordingly, let \( \bar{J}(w, s) \) and \( \bar{H}(w, s) \) be the expected firm and worker surplus, respectively. Then the contract wage maximizes the following Nash product:

\[
\bar{H}(w, s)^\eta \bar{J}(w, s)^{1-\eta},
\]

subject to

\[
w' = \begin{cases} 
  w & \text{with probability } \lambda \\
  w'' & \text{with probability } 1 - \lambda
\end{cases}
\]

\(^{12}\)From GT, to a first order \( \bar{V}_x(s') \) equals the average value for an existing worker \( \bar{V}(s') = \int_w \bar{V}(w', s') dW(w, s) \)

21
where \( \bar{J} \) and \( \bar{H} \) are defined as in (21) and (??).

Given that firms and workers have an approximately similar horizon\(^{13}\), the following first order necessary condition pins down the new contract wage \( w^* \):

\[
\eta \bar{J}(w^*, s) = (1 - \eta) \bar{H}(w^*, s).
\]  

(41)

Given that all renegotiating firms set the same new base wage \( w^* \), we can express the evolution of average base wage across firms \( \bar{w} \) as

\[
\bar{w}' = (1 - \lambda) w^{**} + \lambda \int_w w \frac{1 + x(w, s) + x_r(w, s)}{1 + \bar{x} + \bar{x}_r} dW(w, s).
\]  

(42)

where \( dW(w, s) \) denotes the density function of wages and stocks of workers in temporary unemployment in state \( s \). The last term on the right is the average base wage across firms that are not adjusting wages in the current period. It captures the inertia in wage adjustment.

Let \( w^\dagger(w, s) \) be the expected paycut wage conditional on getting a paycut:

\[
w^\dagger(w, s) \equiv \int_{\gamma^*}^{\gamma^\dagger} w^\dagger(w, \gamma, s) G(\gamma^* - \gamma^\dagger) dG(\gamma),
\]

Then the average firm wage accounting for paycuts is

\[
\bar{w} = \int_w \left[ G(\gamma^\dagger - \gamma^*) w + \left( G(\gamma^* - \gamma^\dagger) \right) w^\dagger(w, s) \right] dW(w, s),
\]  

(43)

where \( G(\gamma^* - \gamma^\dagger) \) is the probability a non-existing firm makes a paycut. The first term on the right is the expected average base wage weighted by the fraction of firms paying the base wage. The second term is the expected paycut wage weighted by the fraction of firms making paycuts.

### 3.5 Households: consumption and saving

We adopt the representative family construct, following Merz (1995) and Andolfatto (1996), allowing for perfect consumption insurance. There is a measure of families on the unit interval, each with a measure one of workers. Before allocating resources to per-capita consumption and savings, the family pools all wage and unemployment income. Additionally, the family owns diversified stakes in firms that pay out profits. The household can then assign consumption \( \bar{c} \) to members and save in the form of capital \( \bar{k} \), which is rented to firms at rate \( r \) and depreciates at the rate \( \delta \).

\(^{13}\)See GT for a discussion of the “horizon” effect in the context of staggered Nash bargaining and of its quantitatively irrelevance.
Let $\Omega$ be the value of the representative household, $\Pi$ profits from the household’s ownership holdings in firms and $T$ are lump sum transfers from the government. Then,

$$\Omega = \max_{\bar{c}, \bar{k}} \{ \log(\bar{c}) + \beta E_t \Omega' \}$$

(44)

subject to

$$\bar{c} + \bar{k} = \bar{w} \bar{n} + b(1 - \bar{n}) + (1 - \delta + r) \bar{k} + T + \Pi$$

and the equation of motion for $\bar{n}$, (4).

The first-order condition from the household’s savings problem gives

$$1 = (1 - \delta + r) E_t \{ \Lambda' \}$$

(45)

where $\Lambda' \equiv \beta \bar{c}/\bar{c}'$.

### 3.6 Resource constraint, government policy, and equilibrium

The resource constraint states that the total resource allocation towards consumption, investment, overhead costs and hiring costs is equal to aggregate output:

$$\bar{y} = \bar{c} + \bar{i} + (\zeta_\gamma \bar{\Gamma} + \zeta_\Theta \bar{\Theta} \bar{G}(\gamma^*) \bar{n} + [\bar{\ell}(x) + \bar{\gamma}_r(x_r)] \bar{G}(\gamma^*) \bar{F}(\vartheta^*) \bar{n}$$

The government funds unemployment benefits through lump-sum transfers:

$$T + (1 - \bar{n}) b = 0.$$

(46)

A recursive equilibrium is a solution for (i) a set of functions $\{J, V, V_r, U\}$; (ii) the hiring rates $x$ and $x_r$; (iii) the recall rate $\bar{p}_r$ and the job finding probability $p$; (iv) the temporary layoff, bankruptcy and paycut thresholds $\theta^*$, $\gamma^*$ and $\gamma^\dagger$; (v) the no-layoffs, no-bankruptcy and no-paycut probabilities $\mathcal{F}(\theta^*)$, $\mathcal{G}(\gamma^*)$ and $\mathcal{G}(\gamma^\dagger)$; (vi) the contract base wage $w^*$; (vii) the paycut wage $\omega^\dagger$; (viii) the subsequent period’s base wage $w'$; (ix) the expected values of the worker- and firm-specific shocks $\Gamma$ and $\Theta$; (x) the averages of $(x, x_r, \vartheta^*, \gamma^*, \gamma^\dagger, \mathcal{F}(\vartheta^*), \mathcal{G}(\gamma^*), \mathcal{G}(\gamma^\dagger), w, \omega^\dagger, \Gamma, \Theta)$; (xi) the rental rate on capital $r$; (xii) the capital labor ratio $\bar{k}$; (xiii) the average consumption and capital $\bar{c}$ and $\bar{k}'$; (xiv) the average employment, temporary and permanent unemployment $\bar{n}$, $\bar{u}_T$, and $\bar{u}_P$. The solution is such that (a) $x$ and $x_r$ satisfy the hiring conditions (22) and (23); (b) $\bar{p}_r$ and $p$ satisfy (6) and (9); (c) $\theta^*$, $\gamma^*$ and $\gamma^\dagger$ satisfy the firm first-order condition (24 ) and the solvency conditions (28) and
(29); (d) $w^*$ satisfies the Nash bargaining condition (41); (e) $\omega^t$ satisfies the solvency condition (30); (f) $w'$ is given by the Calvo process for wages (40); (g) $r$ satisfies the first-order condition for capital renting (25); (h) the rental market for capital clears, that is $\dot{k} = \bar{k} / n^\ddot{c}$; (i) $\bar{c}$ and $\bar{k}'$ solve the household problem; and (j) $\bar{n}$, $\bar{u}_{TL}$, and $\bar{u}_P$ satisfy equations (4), (5) and (1).

4 Model evaluation

In this section we demonstrate the model’s ability to capture the cyclical behavior of hiring, recalls, temporary and permanent layoffs, and “loss of recall” (i.e., transition from temporary to permanent unemployment). We restrict attention to the sample 1978 through 2019. Then in the subsequent section we use the model to study labor market behavior during the Covid-19 recession. We also evaluate the effect of PPP on labor market dynamics.

We first describe the calibration before turning to the results.

4.1 Calibration

We calibrate the model to match moments describing characteristics of temporary layoffs, recall from temporary layoff unemployment, and transitions from temporary layoff unemployment to jobless unemployment; as well as more standard moments describing labor market flows. The model is calibrated to a monthly frequency. There are 16 parameters in the baseline model. We assign 9 of the parameters using external sources. Five of the externally calibrated parameters are common to the macroeconomics literature: the discount factor, $\beta$; the capital depreciation rate, $\delta$; the “share” of labor in the Cobb-Douglas production technology, $\alpha$; and the autoregressive parameter and standard deviation for the total factor productivity process, $\rho_z$ and $\sigma_z$. Our parameter choices are standard: $\beta = 0.99^{1/3}$, $\delta = 0.025/3$, $\alpha = 1/3$, $\rho_z = 0.95^{1/3}$, and $\sigma_z = 0.007$.\textsuperscript{14,15}

Four more parameters are specific to the search literature. We assume a Cobb-Douglas matching function: Our choice of the matching function elasticity with respect to searchers, $\sigma$, is 0.5, the midpoint of values typically

\textsuperscript{14}Note that, in contrast to the frictionless labor market model, the term $\alpha$ does not necessarily correspond to the labor share, since the labor share will in general depend on the outcome of the bargaining process. However, because a wide range of values of the bargaining power imply a labor share just below $\alpha$, here we simply follow convention by setting $\alpha = 1/3$.

\textsuperscript{15}The parameter $\sigma_z$ is chosen to target the standard deviation of output.
used in the literature. We set the worker’s bargaining power $\eta$ to 0.5, as in GT. We normalize the matching function constant, $\sigma_m$, to 1.0. We choose $\lambda$ to target the average frequency of wage changes. Taylor (1999) argues that medium to large-size firms adjust wages roughly once every year; this is validated by findings from microdata by Gottschalk (2005), who concludes that wages are adjusted roughly every year. These observations apply to base pay. Given there are other forms of compensation such as bonuses, we adopt a more conservative value, setting $\lambda = 8/9$, implying an average duration between negotiations of three quarters. The parameter values are given in Table 4.

The remaining parameters are jointly calibrated to match combination of long-run and business cycle moments from the data. We estimate these parameters using a nested, two-stage procedure where we target business cycle moments in an outer loop and long-run moments in an inner loop. In the inner loop, we pick the scale parameter of firm hiring and recall costs, $\chi$; the scale parameters of overhead costs, $\zeta_\gamma$ and $\zeta_\phi$; the exogenous loss-of-recall probability, $1 - \rho_r$; and the flow value of unemployment, $b$; to match long-run flow probabilities and Hall and Milgrom’s (2008) estimate of the relative value of non-employment. The list of parameter values and moments is given in Table 5. In the outer loop, we estimate the parameters dictating the standard deviation of firm- and individual-level costs shocks, $\sigma_\gamma$ and $\sigma_\varphi$, and the hiring and recall elasticities, $\chi/ (\kappa x)$ and $\chi/ (\kappa_r \tilde{x}_r)$. In this step, there are more moments than parameters, and the parameters are estimated to match business cycle moments describing the volatility of separations, hiring, and unemployment. The list of parameter values and targeted moments are given in Tables 5 and 7.

As shown in Table 7, the model is mostly successful in explaining the cyclical volatility of aggregate labor market stocks and flows, with some caveats: for example, the model understates the volatility of separations, and slightly overstates the volatility of jobless unemployment relative to temporary layoff unemployment. Given that we rely on a single driving process to replicate all of the cyclical features of the data, however, we view the fit of the model as more than adequate.

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$^{16}$As in Gertler and Trigari (2009), we interpret the flow value of unemployment $b$ as capturing both unemployment insurance and value of non-work, where the value of non-work includes saved vacancy posting costs.

$^{17}$We normalize the multiplicative means of the distributions of shocks to overhead costs $e^{\mu_e}$ and $e^{\mu_\varphi}$ to unity. We also normalize average productivity to one.
4.2 Results

Next, we explore characteristics of the model further by examining the response of labor market quantities to a one-percent shock to TFP. Figure 4 shows impulse responses for employment, total unemployment, jobless unemployment, temporary layoff unemployment, and the contract wage. The model generates an immediate hump-shaped decrease in total unemployment (and increase in employment). The decrease in total unemployment is somewhat more persistent than generated by similar models, e.g. Gertler and Trigari (2009). This appears to be driven by the slow recovery of jobless unemployment, as temporary layoff unemployment recovers within about two years. That temporary layoff unemployment recovers faster is due to the fact that, everything else equal, (i) costs of recalls are lower than the cost of hiring from the pool of unemployed workers and (ii) some workers from temporary layoff unemployment transition to jobless unemployment.

Figure 5 shows the impulse response of the transition probabilities underlying the dynamic behavior of temporary layoff and jobless unemployment. There are hump-shaped increases for both employment-inflow probabilities; and, consistent with the previous figure, the increase in the probability of moving from jobless unemployment to employment is more persistent than that of moving from temporary layoff unemployment to employment. Both employment outflow probabilities decrease immediately on impact of the shock, but then quickly revert to steady state. Indeed, the probability of moving from employment to jobless unemployment, $p_{EP,t}$, overshoots in its return to steady state. The overshooting property of $p_{EP,t}$ is due to the strong procyclicality of the reservation wage: the annuity value of unemployment in the model is higher during booms. As a result workers are less willing to take paycuts in booms relative to recessions. Hence, while the model generates a countercyclical spike in separations, later on in the expansion exits increase.\footnote{To the extent recessions and booms involve sequences of correlated shocks, however, the model can produce countercyclical separations to permanent unemployment.}

Figure 6 offers a decomposition of the increase in total unemployment from the negative TFP shock into three sources: jobless unemployment, from unemployment spells that begin as jobless unemployment (bottom line); jobless unemployment, but from unemployment spells that begin as temporary-layoff unemployment (difference of middle line and bottom line), and temporary-layoff unemployment, that may yet resolve to recall or jobless unemployment (difference of top and middle line). The figure is analogous to Figure 1, which shows a similar decomposition from the data. Here, the contribution of temporary-layoff unemployment to jobless unemployment is slightly greater than in the
1980s, at around one-half; but substantially less than during the most recent recession.

To demonstrate the extent to which temporary unemployment generates faster employment recoveries, we consider an alternative labor market where workers on temporary layoff immediately move from temporary to permanent unemployment, and thus the number of workers in temporary unemployment is always equal to zero. Under the experiment, firms do not anticipate that workers under temporary unemployment will not be available for recall, and thus put workers on temporary layoff as normal. However, firms correctly understand that no workers from temporary unemployment are available for recall. Figure 4 shows the difference in labor market stocks across the baseline and no-$u_{TL}$ economies. Figure 5 shows the difference in labor market flows. As can be seen, the initial fall in employment is lower under the no-$u_{TL}$ economy, but the recovery is employment is longer, with a gap in employment recoveries that grows over time. Firms face quadratic adjustment costs when incorporating new hires into the firm, and thus are constrained by the number of new hires they can absorb into their existing labor force at a given point in time. The growing gap in employment recoveries across the baseline and no-$u_{TL}$ economies demonstrates that the firm’s reliance on new hires to expand the labor force — as opposed to both new hires and recalls — generates aggregate employment losses that cumulate over time.

Note, the difference in employment recoveries shown in Figure 4 might suggest a minor role of temporary unemployment for generating faster employment recoveries. However, the labor market dynamics show in Figure 4 are generated under a steady state that averages periods where temporary unemployment played a large role for unemployment dynamics (i.e. the 1980s recessions) with periods where the role of temporary unemployment was decidedly less pronounced (i.e., the Great Recession). As we show in the next section, temporary unemployment plays a more important role for labor market dynamics over periods where it comprises a greater share of total unemployment.

5 Unemployment dynamics during the Covid-19 recession and the role of the Paycheck Protection Program

As we have discussed, a signature feature (and anomaly) of the labor market during the recent recession was the immediate and unprecedented sharp flow
of workers from employment to temporary layoff. In this section we adapt our model to capture the dynamics of unemployment during this period, with particular emphasis on the interaction between temporary layoff and jobless unemployment.

As we have also discussed, another distinctive feature of the labor market was the introduction of the Payroll Protection Program, which was the largest component of the federal government’s initial fiscal response to the pandemic. The program was intended not only to encourage business to rehire workers from temporary layoff, but also to keep temporary-layoff unemployment “temporary,” by staving off a massive exit of firms to bankruptcy. Thus, we apply our model to evaluate the effectiveness of PPP. To do so, we first adapt our framework to explain the dynamics of temporary layoff versus jobless unemployment over the pandemic, taking into account PPP. We then do a counterfactual exercise where PPP is removed.

We do not model the endogenous spread of the virus. Instead we capture the economic consequences of the pandemic through two types of exogenous shocks: First, we introduce “lockdown” shocks whereby workers from employment move to temporary-layoff unemployment. Second, we interpret the economic disruption resulting from the pandemic as negative capital and labor utilization shocks that manifest as shocks to effective TFP. We then rely on the structure of the model to study the labor market response to the pandemic and PPP as endogenous responses to shocks to economic fundamentals. Finally, after we estimate the series of shocks that capture the economic disturbances owing to the pandemic, we study how the labor market would have responded in the absence of PPP.

5.1 Adapting the model

Here we describe a few modifications introduced to adapt the model to the pandemic recession. We begin by discussing the two shocks in the model introduced to capture the direct effect of the pandemic on the economy: “lockdown” shocks, which move workers from employment to temporary-layoff unemployment; and shocks to effective TFP, capturing disruption to factor utilization arising from social distancing, either through formal restrictions or voluntary aversion to the virus.

We assume that lockdown shocks are $i.i.d.$ unanticipated shocks realized at the beginning of a period that hit a fraction $1 - \nu$ of a firm’s labor force. The fraction $1 - \eta$ of workers in the firm who are hit by the lockdown shock and were either employed or recalled by the firm in the previous period are sent to temporary layoff. Workers hit by the lockdown shock who were new
hires in the previous period return to jobless unemployment. Thus, the law of motion for employment becomes

\[ \bar{n}' = \nu (1 + \bar{x} + \bar{x}_r) \bar{G}(\gamma^*) \bar{F}(\psi^*) \bar{n}. \] (47)

Note that though the lockdown shock is i.i.d., it will have persistent effects since it takes time for workers laid off to return to employment.

Workers in lockdown are indistinguishable from other workers in temporary-layoff unemployment, except that they move exogenously from temporary-layoff unemployment to jobless unemployment at a potentially different rate, \(\rho_{r\phi}\). Here we allow for the possibility that workers separated from the firm due to the pandemic may have a different degree of attachment to the firm than the typical worker put on temporary unemployment.

Accordingly, the law of motion for temporary-layoff unemployment becomes

\[ \bar{u}'_{TL} = \bar{G}(\gamma^*)(1 - \bar{p}_r) \left( \bar{\phi} \rho_r + (1 - \bar{\phi}) \rho_{r\phi} \right) \bar{u}_{TL} \]

\[ + \left[ \nu (1 - \bar{F}(\psi^*)) \bar{G}(\gamma^*) + (1 - \nu)(1 - \eta) \bar{G}(\gamma^*) \right] \bar{n}, \] (48)

where \(1 - \phi\) denotes the fraction of workers in temporary-layoff unemployment who are on lockdown. As such, the law of motion for the number of workers under lockdown is given by

\[ (1 - \phi') \bar{u}'_{TL} = (1 - \nu)(1 - \eta) \bar{G}(\gamma^*) \bar{n} + \bar{G}(\gamma^*)(1 - \bar{p}_r) \rho_{r\phi}(1 - \phi) \bar{u}_{TL}. \] (49)

We also allow for the possibility that it is less costly to recall workers on lockdown than other workers from temporary layoff. In particular, we assume that the adjustment component of recall costs to the firm are reduced by a term proportional to the fraction of workers in a firm who are on lockdown:

\[ \iota_r(x_r) = \chi x_r + \frac{\kappa_r}{2} \left( x_r - \xi \frac{(1 - \phi) u_{TL}}{\bar{F}(\psi^*) \bar{n}} \right)^2, \] (50)

where \(0 < \xi < 1\).

The parameters \(\xi\) and \(\rho_{r\phi}\) represent the only changes to the baseline structural model presented in the second section of the paper. Both are estimated from the data.

As discussed in section 2.21, we model “social distancing” effects on productivity via the impact on capital and labor utilization, respectively \(\xi_k\) and \(\xi_n\). From equation(11) effective total factor productivity \(z\) depends on “true TFP” \(\tilde{z}\) as well as \(\xi_k\) and \(\xi_n\) as follows:

\[ z = \tilde{z} \xi_k \xi_n^{1-\alpha} \] (51)
We assume for the pandemic exercise that $\tilde{z}$ is fixed but that $\xi_k$ and $\xi_n$ vary in a way that has $z$ obey the following first order process:

$$\log z' = \rho_z \log z + \varepsilon_z$$  \quad (52)$$

When then suppose that over the pandemic there are three negative realizations of the shock $\varepsilon_z$, each at a point where the pandemic accelerated: April 2020, September 2020 and January 2021. We estimate $\rho_z$ directly from the data as well as the sizes of each of the three shocks to $\varepsilon_z$.

We treat PPP as a direct factor payment subsidy $\tau$ to the firm, similar to Kaplan, Moll, and Violante (2020). The period output that enters the firm’s value of a unit of labor changes accordingly, to $(1 + \tau)zF(\vartheta^*)\tilde{k}^\alpha$. Hence, an economy-wide reduction in utilization $z$ can be counteracted by a forgivable loan from PPP. Anecdotal evidence suggests that funds from PPP were spent immediately; and indeed, the appropriated funds for each of the three rounds of PPP were exhausted soon after they were made available. Thus, we assume that the majority of PPP funds are spent as they are allocated, as we make clear in the next section.

5.2 Simulating the pandemic recession

5.2.1 Model implementation

We initialize the model from a January 2020 steady state. We then estimate the model so that we match labor market data from the CPS. We date the start of the pandemic recession in March 2020 when the labor market started to weaken. Given the dispersed timing in the geographic spread of the pandemic, we allow the $i.i.d.$ lockdown shock to hit each month, beginning in March. We allow for three major persistent utilization shocks, corresponding to periods where the pandemic quickly accelerated, occurring in April 2020, September 2020 and January 2021. For April 2020, further, we allow an additional transitory utilization shock to hit as well. We think of the transitory shock as capturing a one-time disruptions to economic activity that occurred at the beginning of the pandemic. The estimation pins down the relative importance of the persistent and transitory shocks.\(^{19}\)

We implement PPP given the following considerations. The first two rounds of PPP overlapped and amounted to roughly 659 billion dollars, about 12.5 of quarterly GDP. The third round of PPP amounted to roughly 5.4% of

\(^{19}\)As a practical matter, the April 2020 utilization shock is the largest to hit. We are effectively allowing the persistence of this shock to differ from the two others.
quarterly GDP. We thus calibrate the total amount of the first two rounds of PPP within the model as 12.5% of quarterly steady state output and the third round of PPP as 5.4% of quarterly steady state output. Finally, PPP was designed to be delivered to businesses as a forgiveable loan; and as of January 2021, 85% of applications for loan forgiveness have been approved. Hence, we treat the 85% of the total amount of PPP as a production subsidy.

Although legislation for the first round of PPP was introduced at the end of March 2021, the first month of PPP was hectic and characterized by confusion over eligibility for the program. It unlikely that the effects of PPP would be seen by the second week of April (when we observe labor market data for the month from the CPS). Thus, we allow implementation of PPP in the model to begin in May 2021. Funding from the first two rounds of PPP ran out by the beginning of August. We assume that the majority of the first two rounds of PPP is paid as equal sums for the months of May, June, and July in 2020. We assume that a small remainder of the original allocation is paid out in amounts that decline geometrically at rate \( 1 - \rho_r = 1 - (0.25)^{1/3} = 0.37 \). The first two rounds of PPP are announced the date of implementation, after which the associated sequence of disbursements is anticipated by agents in the economy.

The third (and final) round of PPP totals 284 billion dollars and was authorized at the end of December 2020. The program ran out of money at the beginning of May 2021. Thus, we assume in the model that the funds associated with the third round are paid out in equal sums in January, February, March, and April 2021. The remainder of the allocation is paid out in sums that decline geometrically at rate \( 1 - \rho_r \). Similar to the first two rounds, the final round of PPP is announced the date of implementation, and the entire sequence of disbursements is anticipated after announcement.

5.2.2 Estimation with labor market data

After calibrating the model to a January 2020 steady state, we estimate the model to match data through June 2021.\(^{20}\) We estimate: the two additional model parameters \( \xi \) and \( \rho_{\phi} \); the autoregressive coefficient for the persistent utilization shocks \( \rho_z \); the sizes of the monthly \( i.i.d. \) lockdown shocks; and the sizes of the three persistent utilization shocks, as well as the size of the April 2020 transitory utilization shock. We estimate the model to match monthly levels of temporary layoff and jobless unemployment; gross flows from temporary layoff unemployment to permanent layoff unemployment; and gross flows

\(^{20}\) Note, although February 2020 is the start of the official NBER recession, we observe no appreciable changes in labor market quantities or flows for this month. Hence, we do not target labor market stocks or flows associated with this month.
from temporary layoff unemployment to employment. We also include gross flows from employment to permanent unemployment from March to April as a target.

For gross flows from temporary layoff to jobless unemployment $g_{t}^{T, JL}$ in the quarter starting in April 2020, we target total gross flows over the quarter rather than monthly gross flows. Over this time period, monthly gross flows from temporary layoffs to permanent unemployment exhibit hump-shaped behavior. We suspect that some of this is due to peculiarities in the survey structure of the CPS. Thus, rather than forcing the model to match the monthly $g_{t}^{T, JL}$ gross flows for these three months, we have the model match total gross flows over the three-months period.

Thus, we estimate three parameters ($\xi$, $\rho_{r\phi}$, and $\rho_{z}$) and nineteen shocks (three persistent utilization shocks, one transitory utilization shock, and fifteen i.i.d. lockdown shocks) to match 59 moments from the data. Hence, the system is overidentified.

Identification of model parameters and shocks can be understood as follows: the parameter $\xi$ dictates the cost of recalling workers from temporary unemployment back to employment, and thus is informed by the sequence of gross flows from temporary layoff unemployment to employment, $g_{t}^{T, LE}$. The parameter $\rho_{r\phi}$ dictates the rate at which workers exogenously lose their recall option in temporary layoff unemployment and go to jobless unemployment, and thus is informed by the sequence of gross flows $g_{t}^{T, JL}$. Conditional on matching the sequence of $g_{t}^{T, LE}$ and $g_{t}^{T, JL}$ gross flows, the lockdown shocks $\{1 - \nu_{\tau}\}_{\tau=0}^{14}$ are informed by the level of temporary unemployment. The three persistent utilization shocks are informed by the time series of permanent unemployment. Finally, the transitory utilization shock only affects endogenous separations, as hiring is purely forward looking; thus, the one-time transitory utilization shock is identified by the spike in gross flows from employment to permanent unemployment $g_{t}^{E, JL}$ from March to April.

Estimates of the three parameters are given in Table 8. Estimates of the three persistent utilization shocks and the one-time transitory utilization shock are given in Table 9. The full series of shocks (including PPP) and the endogenous dynamics for the fraction of workers in temporary unemployment on lockdown are given in Figure 9. Several characteristics of the estimates are striking. First, note that the estimated value of $\rho_{r\phi}$ is higher than $\rho_{r}$. This indicates that workers in temporary unemployment due to lockdown move to jobless unemployment at a lower rate than workers in temporary layoff unemployment due to endogenous layoff. Then, note that $\xi$ is equal approximately to one half. This indicates that, while it was easier to recall workers in temporary layoff unemployment due to lockdown than other workers in temporary
layoff unemployment, reversing the initial flow of workers from employment to temporary layoff did not come for free.

5.3 Results

5.3.1 The pandemic labor market

Figure 10 shows the estimated series for employment, temporary-layoff unemployment, jobless unemployment, and total unemployment against the data. The model does well at matching the initial rise in temporary-layoff unemployment and the accompanying reduction in total employment. The reduction in employment in the model is only slightly less than from the data, and the model is successful at generating the persistent reduction in employment that continues through the second quarter of 2021. Jobless unemployment in the data jumps from just above three percent to around six percent from March to April, and then remains steady until gradually falling in February 2021. Jobless unemployment from the model tracks the data rather well, although the model generates a slightly higher level at the plateau. The panel for total unemployment shows that the model does fairly well at matching the composite series from the data.

Figure 11 shows the estimated series for gross flows in the data against those from the model. Gross flows from employment to temporary layoff unemployment $g_{E,TL}^t$ jump to nearly 0.15 in April of 2020, and thereafter stay above one percent until January of 2021. The model is successful in matching this pattern from the data through the estimated lockdown shocks. What is perhaps more interesting is the behavior of $g_{TL,E}^t$, which is dictated by firms’ recalls of workers in temporary-layoff unemployment; and $g_{T,JO}^t$, which is generated through the endogenous and exogenous forces that sever the link between workers in temporary unemployment and firms. As can be seen by the figure for $g_{TL,E}^t$, gross flows from temporary unemployment to employment immediately increase in May and June. The model is largely successful at matching the time series for $g_{TL,E}^t$. Both in the model and in the data, gross flows from temporary unemployment to employment hit their peak in June of 2020. In the model, this comes after the realization of the persistent utilization shocks (in April) and the release of PPP funds (in May). Given that the negative utilization shock and PPP move recall behavior in opposite

\[21\] Gross flows $g_{AB,t}$ from $A$ to $B$ at time $t$ are constructed as the number of workers in $A$ at time $t-1$ who are observed at $B$ at time $t$. In both the data and the model, the size of the labor force is normalized to unity. Hence, if $g_{AB,t} = 0.05$, a number of workers equal to 5% of the labor force move from $A$ to $B$ from $t-1$ to $t$.
directions, this suggests the importance of PPP for generating the observed flow of recalls.

Both the data and the model show an immediate increase in gross flows from temporary layoff to jobless unemployment $g_{t}^{TL, JL}$ after May 2020. This comes in spite of a reduction in the observed probability of workers from temporary unemployment moving to permanent unemployment, as pointed out by Hall and Kudlyak (2020) and shown in Figure A.5 of the Appendix. Note, gross flows $g_{t}^{TL, JL}$ are the product of temporary unemployment, $u_{TL}$, and the probability of moving from temporary layoff to jobless unemployment, $g_{t}^{TL, JL}$. Thus, even though the probability of moving from temporary layoff to jobless unemployment decreases, the rise in temporary layoff unemployment is large enough that overall flows from temporary to permanent unemployment increase. However, the magnitude of such flows always remains below one percent of the total labor force. The model is able to match the overall pattern of gross flows from temporary layoff to permanent unemployment in the data through the lower probability that workers in lockdown move exogenously to temporary unemployment (compared to workers in temporary unemployment from endogenous temporary layoff); and through the dynamics of $1 - \phi_t$, the fraction of workers in temporary unemployment due to lockdown.

The model generates the sudden rise in flows from employment to jobless unemployment $g_{t}^{E, JL}$ seen in the data, as well as the sudden drop in flows from permanent unemployment to employment $g_{t}^{JL, E}$. Beginning in the summer of 2020, the model predicts lower $g_{t}^{E, JL}$ and $g_{t}^{JL, E}$ flows than are seen in the data. However, these are offsetting flows, and so the model is still successful at generating the plateau in jobless unemployment shown in the previous figure. Put differently, the model matches the net flows between employment and jobless unemployment.

5.3.2 Assessing the role of PPP

Overall, the model appears successful at matching the dynamic behavior of labor market stocks and flows during the recent recessions. It is thus a credible framework to evaluate the impact of PPP in affecting labor market activity. To do so, we solve the full equilibrium labor market dynamics implied by the model under the same sequence of lockdown and utilization shocks estimated from the data, but with no transfers due from PPP. Thus, we study whether PPP was successful at preserving connections between firms and workers and in encouraging firms to recall workers on lockdown in temporary unemployment.

Figure 12 shows the behavior of labor market stocks in the pandemic labor market for the baseline model and a counterfactual without PPP. The no-
PPP counterfactual shows larger and more persistent employment reductions than under the baseline. For example, whereas employment in August 2020 is 6.8 percentage points below pre-pandemic levels under the baseline model, employment in August 2020 is instead 9.3 percentage points below the pre-pandemic level under the no-PPP counterfactual.

Temporary layoff unemployment is slightly higher under the no-PPP counterfactual; but the bulk of the difference in employment levels comes from a greater number of workers in jobless unemployment. Jobless unemployment hits 8.5% in May of the no-PPP counterfactual (compared to 6.5% of the baseline model) and remains persistently higher through the spring of 2021. The difference in employment across the baseline and counterfactual labor markets only shrinks below a percentage point by June 2021. The baseline and no-PPP counterfactuals converge by September of 2021. Interestingly, both models predict a relatively slow recovery of employment, with employment still about a percentage point and a half below its pre-pandemic level in October 2021.

Figure 13 shows the difference in gross flows under the baseline model and no-PPP counterfactual, identifying the source of the different employment levels across the baseline and counterfactual labor markets. We see immediately that the better labor market performance under the baseline model with PPP is due to a larger number of recalled workers, observed in the reduction of gross flows from temporary unemployment to employment $g_{t}^{TL,E}$ in no-PPP counterfactual. This is unsurprising: the value of an employed workers is lower given the persistent shock to utilization from the pandemic. Without PPP to mitigate the production impact of the pandemic, firms have less reason to recall workers in temporary unemployment on lockdown.

While a decline in gross flows from temporary unemployment to employment $g_{t}^{TL,E}$ under the no PPP counterfactual can be understood as the source for lower employment levels, this alone is not sufficient to understand the behavior of stocks shown in the previous figure, where lower employment levels are accounted for by a higher permanent unemployment. Thus, the difference in gross flows from temporary to permanent unemployment $g_{t}^{TL,JL}$ across the baseline model and the no-PPP counterfactual are informative: Flows from temporary to permanent are more than twice as large for multiple periods. Under the model structure, this is intuitive: the more periods a worker in temporary unemployment is not recalled, the likelier it is that the worker is reallocated from temporary to permanent unemployment, either due to job destruction or exogenous removal from temporary unemployment. Thus, the model illustrates that the labor market is not capable of maintaining an arbitrary number of workers in temporary unemployment for any given duration.
Hence, part of the success of PPP was the speed with which it was formulated and executed, enabling firms to quickly recall workers and scale back temporary layoffs.

Turning to flows between employment and permanent unemployment, we see that gross flows from permanent unemployment to employment $g_{t}^{JL,E}$ are substantially lower under the no-PPP counterfactual, whereas gross flows from employment to permanent unemployment $g_{t}^{E,JL}$ look largely similar. The lower levels of $g_{t}^{JL,E}$ flows explains how employment is persistently lower under the no-PPP counterfactual.

Taken as a whole, our estimates imply that PPP was successful in fulfilling its intended purpose of encouraging firms to rehire workers on temporary layoff. The cumulative number of workers moving from temporary to permanent unemployment from May to September 2020 is 48.0% of what it would have been without PPP. Cumulative recalls from temporary unemployment over the same period are roughly double what they would have be without PPP. We estimate an average monthly increase in employment of around 2.14% over the same period, roughly consistent with estimates from Hubbard and Strain (2020). The estimated model attributes persistent employment gains to PPP of at least one percentage point through May 2021.

Finally, we study the role of wage stickiness in generating an employment effect of PPP. Part of the logic of PPP as a payroll subsidy appears predicated on the notion that prevailing wages constrain firms from recalling workers from temporary unemployment. To probe this interpretation, we introduce the estimated shocks into a variant of the baseline model with flexible wages (via period-by-period wage bargaining).

Figure 15 shows the changes in stocks due to PPP under the staggered contracting baseline and under period-by-period contracting. Under the model with flexible wages, PPP generates a smaller immediate employment recovery, with a modest boost to average monthly employment of around 1.5% over the period May to September 2020. As is shown in the Figure, PPP achieves a slightly larger reduction in temporary unemployment under the baseline case of multiperiod bargaining compared to period-by-period bargaining; and a substantially larger reduction in permanent unemployment under multiperiod bargaining.

However, the stocks tell only part of the story: Figure 16 shows the changes in select gross flows from PPP under staggered versus period-by-period contracting. Here, we see that PPP generates a stronger response in recalls under staggered contracting, as seen by the $g_{TL,E}$ flows; and that PPP also prevents more workers in temporary unemployment from moving to permanent unemployment with staggered contracting, as seen by the greater reduction
in $gL_{L,JL}$ flows. Indeed, cumulative recalls from temporary unemployment from May to September 2020 are just 12.0% higher due to PPP under flexible wages, compared to 97.3% higher under the baseline model with staggered contracting; and the cumulative reduction in the number of workers moving from temporary to permanent unemployment drops by just 12.3 with flexible wages, compared to the 48% reduction achieved under the baseline model with wage stickiness.

Hence, the extra reduction in permanent unemployment from PPP under staggered contracting comes by preserving existing jobs, achieved directly through higher recalls and indirectly by allowing fewer workers to move from temporary to permanent unemployment. Thus, a structural evaluation of PPP abstracting from wage stickiness will miss the full extent to which the program fulfilled its mandate in preserving existing jobs.

6 Conclusion

This paper develops a quantitative model of unemployment dynamics that distinguishes between temporary and permanent layoffs. Our main motivation is to understand how the extraordinary increase in temporary layoffs during the pandemic affected overall labor market behavior and also how the Paycheck Protection Program may have worked to facilitate the return of workers to employment from temporary layoff. We note also that, though of varying importance, temporary layoffs have played a role in most postwar recessions. Thus having a model with both temporary and permanent unemployment may be of interest beyond the current recession.

In our model, employment separations are endogenous and firms decide whether workers are put on temporary or permanent unemployment. Firms also decide about hiring workers from permanent unemployment or recalling from temporary unemployment. We calibrate the model to data prior to the recession, specifically over the sample 1979 to 2019. We show that the model fits labor market dynamics over this period. We then adapt the model to the current recession and show that it can also fit this period well. In doing so, we allow for the fact that PPP was in place. We then examine the effect of PPP by removing it and re-simulating the model. We find that without PPP permanent unemployment would have been persistently higher and employment persistent lower. Without PPP, firms would have recalled fewer workers from temporary unemployment and more workers on temporary layoff would have drifted to permanent unemployment.
References


Table 1: Unemployment, permanent unemployment, and temporary unemployment

<table>
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<th>$U = J_L + T_L$</th>
<th>$J_L$</th>
<th>$T_L$</th>
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<tr>
<td>mean($x$)</td>
<td>0.062</td>
<td>0.054</td>
<td>0.008</td>
</tr>
<tr>
<td>std($x$)/std($Y$)</td>
<td>8.518</td>
<td>8.532</td>
<td>10.906</td>
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<td>corr($x$, $Y$)</td>
<td>-0.848</td>
<td>-0.810</td>
<td>-0.788</td>
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For second and third row, series are taken as (1) quarterly averages of seasonally adjusted monthly series, (2) logged, then (3) HP-filtered with smoothing parameter of 1600

Table 2: Gross worker flows, 1978:I–2019:IV

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
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<tbody>
<tr>
<td>$E$</td>
<td>$E$</td>
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<tr>
<td>0.984</td>
<td>0.005</td>
</tr>
<tr>
<td>$T_L$</td>
<td>0.482</td>
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<tr>
<td>$J_L$</td>
<td>0.304</td>
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Series are quarterly averages of seasonally-adjusted monthly series

Table 3: Assigned parameters

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>$\beta$ 0.997 = 0.99^{1/3}</th>
<th>$\delta$ 0.008 = 0.025/3</th>
<th>$\alpha$ 0.33</th>
<th>$\rho_z$ 0.99^{1/3}</th>
<th>$\sigma_z$ 0.007</th>
<th>$\sigma$ 0.5</th>
<th>$\eta$ 0.5</th>
<th>$\sigma_m$ 1.0</th>
<th>$\lambda$ 8/9 (3 quarters)</th>
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42
### Table 4: Jointly calibrated parameters and targeted moments, inner loop

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
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<tr>
<td>$\chi$</td>
<td>Scale, hiring costs</td>
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<td>Average $JL \rightarrow E$ rate (0.304)</td>
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<td>$\varsigma_\theta \cdot e^{\mu_\theta}$</td>
<td>Scale, overhead costs, worker</td>
<td>0.0893</td>
<td>Average $E \rightarrow T$ rate (0.005)</td>
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<td>$\varsigma_\gamma \cdot e^{\mu_\gamma}$</td>
<td>Scale, overhead costs, firm</td>
<td>2.0097</td>
<td>Average $E \rightarrow JL$ rate (0.011)</td>
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<tr>
<td>$1 - \rho_r$</td>
<td>Loss of recall rate</td>
<td>0.3925</td>
<td>Average $T \rightarrow JL$ rate (0.210)</td>
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<td>$b$</td>
<td>Flow value of unemployment</td>
<td>0.8848</td>
<td>Relative value of non-work (0.71)</td>
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### Table 5: Jointly calibrated parameters, outer loop

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<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi/(\kappa \tilde{x})$</td>
<td>Hiring elasticity, new hires</td>
<td>0.3942</td>
</tr>
<tr>
<td>$\chi/(\kappa_r \tilde{x}_r)$</td>
<td>Hiring elasticity, recalls</td>
<td>0.8912</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>Parameter lognormal $\mathcal{F}$</td>
<td>1.1410</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>Parameter lognormal $\mathcal{G}$</td>
<td>0.3215</td>
</tr>
</tbody>
</table>

### Table 6: Targeted moments, outer loop

<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of hiring rate</td>
<td>3.304</td>
<td>3.253</td>
</tr>
<tr>
<td>SD of total separation rate</td>
<td>6.620</td>
<td>4.707</td>
</tr>
<tr>
<td>SD of temporary-layoff unemployment, $u_{TL}$</td>
<td>10.906</td>
<td>10.969</td>
</tr>
<tr>
<td>SD of jobless unemployment, $u_{JL}$</td>
<td>8.532</td>
<td>10.519</td>
</tr>
<tr>
<td>SD of hiring rate from jobless unemployment relative to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of recall hiring rate from temporary-layoff unemployment</td>
<td>0.445</td>
<td>0.442</td>
</tr>
</tbody>
</table>
Table 7: Estimated parameters for pandemic experiment

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_z$</td>
<td>Autoregressive coefficient for persistent utilization shocks</td>
<td>0.7651</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Adjustment costs for workers on lockdown</td>
<td>0.4988</td>
</tr>
<tr>
<td>$1 - \rho_{r\phi}$</td>
<td>Probability of exogenous loss of recall for workers in temporary-layoff unemployment</td>
<td>0.6329</td>
</tr>
</tbody>
</table>

Table 8: Estimated shocks for pandemic experiment

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistent utilization shock, April 2020</td>
<td>$-10.28%$</td>
</tr>
<tr>
<td>Transitory utilization shock, April 2020</td>
<td>$-0.90%$</td>
</tr>
<tr>
<td>Persistent utilization shock, September 2020</td>
<td>$-4.23%$</td>
</tr>
<tr>
<td>Persistent utilization shock, January 2021</td>
<td>$-9.56%$</td>
</tr>
</tbody>
</table>
The figure shows a decomposition of the stock of unemployed workers across various recessions. The top (solid) line shows the total unemployment rate, taken across all workers in unemployment. The middle (dashed) line shows the unemployment rate that only includes workers in jobless unemployment. The bottom (dash-dotted) line further subtracts workers in jobless unemployment who entered unemployment through temporary layoff. Thus, the difference of the middle and bottom lines shows the contribution of temporary layoffs to jobless unemployment (via workers previously in temporary-layoff unemployment who lose connection to their previous employer); the difference of the top and middle lines shows the contemporaneous contribution of workers in temporary-layoff unemployment to total unemployment; and the difference of the top and bottom lines shows the entire contribution of temporary unemployment-layoff to total unemployment.
Figure 2: Total and temporary unemployment across sectors, COVID-19 Recession

Figure 3: Stocks and flows
Figure 4: TFP shock, responses of employment, unemployment, and wages
Figure 5: TFP shock: responses of labor market transition probabilities
The figure offers a decomposition of the increase in total unemployment from three sources: jobless unemployment, from unemployment spells that begin as jobless unemployment (bottom line); jobless unemployment, but from unemployment spells that begin as temporary-layoff unemployment (difference of middle line and bottom line), and temporary-layoff unemployment, that may yet resolve to recall or jobless unemployment (difference of top and middle line).
Figure 7: TFP shock, no temporary unemployment: responses of employment, unemployment, and wages
Figure 8: TFP shock, no temporary unemployment: responses of labor market transition probabilities

Figure 9: COVID-19 lockdown, shocks
Figure 10: COVID-19 lockdown, stocks
Figure 11: COVID-19 lockdown, gross flows
Figure 12: No PPP counterfactual, stocks
Figure 13: No PPP counterfactual, flows
Figure 14: Changes in stocks due to PPP, sticky vs. flex wages
Figure 15: Change in gross flows involving temp. unempl. due to PPP, sticky vs. flex wages
A Appendix materials

A.1 Reclassifying workers

There are several discrepancies with self-reported employment statuses after the onset of Covid-19 pandemic. First, as noted by the BLS, workers who should have been classified as being on temporary layoff instead were classified as absent from work without pay (U.S. Bureau of Labor Statistics, 2020). Second, at the beginning of the pandemic, there was an unusually large flow of workers moving from employment to out-of-the-labor-force (OLF) but willing to take a job at the beginning of the pandemic; i.e. workers moved directly from employment to becoming non-employed “discouraged workers”. The flow is particularly large for workers who move to OLF and are not searching for stated reasons including that they believe that there is no work available in their area of expertise, that they could not find work, or for reasons classified as “other”.

The approach that we take to correct for these issues is motivated by Figure 6 (and the discussion thereof) from a speech given by Jerome H. Powell at the Economic Club of New York on February 10, 2021. However, we want to correct not just erroneous stocks, but also erroneous flows, which makes the correction slightly more involved.

Consider a month $t$, where we observe $N_t$ workers. Each worker is classified into one of four different employment states, encoded in a variable $Status_{it}$:

- $\tilde{E}_t$, employed
- $\tilde{T}_t$, temporary unemployed
- $\tilde{P}_t$, permanent unemployed
- $\tilde{I}_t$, inactive

Two subsets of the groups above are misclassified

- A fraction $x_{E_{wop},t}$ of $E_{wop,t} \subset \tilde{E}_t$ (employed without pay) should be classified as “temporary unemployed” in month $t$
- A fraction $x_{I_{dis},t}$ of $I_{dis,t} \subset \tilde{I}_t$ (inactive but discouraged) should be classified as “permanent unemployed” in month $t$

To obtain the scalars $x_{E_{wop},t}$ and $x_{I_{dis},t}$, we attribute increases in $E_{wop,t}$ and $I_{dis,t}$ after February 2020 to response error.
Next, let $n_t^Z$ denote the number of workers in state $Z_t$. Then, we have

\[
\begin{align*}
    n^E_t &= (1 - x_{E_{wop},t}) \cdot n^E_t \\
    n^T_t &= n^T_t + x_{E_{wop},t} \cdot n^E_t \\
    n^P_t &= n^T_t + x_{I_{dis},t} \cdot n^I_t \\
    n^I_t &= (1 - x_{I_{dis},t}) \cdot n^I_t \\
\end{align*}
\]

To compute corrected flows, we follow the steps below:

- First, define the following quantities:

\[
\begin{align*}
    E_{-,t} &= \tilde{E}_t - E_{wop,t} \\
    I_{-,t} &= \tilde{I}_t - I_{dis,t} \\
\end{align*}
\]

- Compute flows between

\[
\begin{align*}
    \{E_{-,t}, E_{wop,t}, T_t, P_t, I_{-,t}, I_{dis,t}\} \\
\end{align*}
\]

\[
\begin{align*}
    \{E_{-,t+1}, E_{wop,t+1}, T_{t+1}, P_{t+1}, I_{-,t+1}, I_{dis,t+1}\} \\
\end{align*}
\]

Denote the number of flows between two states $Z_t$ and $W_{t+1}$ as $n^{Z,W}_{t,t+1}$. For example, compute $n^{E_{-,t}}_{t,t+1}$ as

\[
\begin{align*}
    n^{E_{-,t}}_{t,t+1} &= \sum_{i \in E_{-,t} \cap \tilde{T}_{t+1}} i \\
\end{align*}
\]

- Then, for $Z_t \in \{E_{-,t}, E_{wop,t}, I_{-,t}, I_{dis,t}, \tilde{P}_t, \tilde{T}_t\}$, compute

\[
\begin{align*}
    n^{Z,E}_{t,t+1} &= n^{Z,E}_{t,t} + (1 - x_{E_{wop},t+1}) \cdot n^{Z,E}_{t+1} \\
    n^{Z,T}_{t,t+1} &= n^{Z,T}_{t,t} + (1 - x_{I_{dis},t+1}) \cdot n^{Z,I_{dis}}_{t+1} \\
    n^{Z,P}_{t,t+1} &= n^{Z,P}_{t,t} + x_{I_{dis},t+1} \cdot n^{Z,I_{dis}}_{t,t+1} \\
    n^{Z,T}_{t,t+1} &= n^{Z,T}_{t,t} + x_{E_{wop},t+1} \cdot n^{Z,E}_{t,t+1} \\
\end{align*}
\]

- For $Z_{t+1} \in \{E_{t+1}, I_{t+1}, P_{t+1}, T_{t+1}\}$, compute

\[
\begin{align*}
    n^{E,Z}_{t,t+1} &= n^{E,Z}_{t,t+1} + (1 - x_{E_{wop},t}) \cdot n^{E,Z}_{t+1} \\
    n^{I,Z}_{t,t+1} &= n^{I,Z}_{t,t+1} + (1 - x_{I_{dis},t}) \cdot n^{I,Z}_{t+1} \\
    n^{P,Z}_{t,t+1} &= n^{P,Z}_{t,t+1} + x_{I_{dis},t} \cdot n^{P,Z}_{t,t+1} \\
    n^{T,Z}_{t,t+1} &= n^{T,Z}_{t,t+1} + x_{E_{wop},t} \cdot n^{T,Z}_{t,t+1} \\
\end{align*}
\]
• Then,

\[ n_t^Z = n_{t,t+1}^{Z,E} + n_{t,t+1}^{Z,I} + n_{t,t+1}^{Z,P} + n_{t,t+1}^{Z,T} \]

and

\[ p_t^{Z,W} = \frac{n_{t,t+1}^{Z,W}}{n_t^Z} \]

See Figure A.3 for a comparison of the raw and adjusted stocks and flows.
A.2 Estimating the fraction of permanent unemployed who exited employment via temporary layoff

We want to calculate the number of workers whose most recent exit from employment was to temporary-layoff unemployment; but who are currently in jobless unemployment.

First, consider workers whose most recent exit from employment was to temporary-layoff unemployment, across dates $t - m - 1$ and $t - m$. Denote 

$$x_{t-m-1, t-m} = e_{TL} \cdot \left( n_{t-m-1}^{E} \cdot p_{t-m}^{E, TL} \right)$$

to be the $t - m$ distribution of workers who most recent exit from employment was to temporary-layoff unemployment, occurring between periods $t - m - 1$ and $t - m$; where $e_{TL}$ is a column vector with an entry of one in the $TL$'th place and zeros elsewhere. Note, $p_{t-m}^{E, TL}$ is the probability of moving from employment to temporary layoff unemployment at time $t - m$; and hence, $n_{t-m-1}^{E} \cdot p_{t-m}^{E, TL}$ is the number of workers moving from employment to temporary layoff unemployment at time $t - m$. Although the distribution $x_{t-m-1, t-m}$ is degenerate and concentrated in state $TL$ at time $t - m$, this will not be the case in future periods.

We wish to track the movement of workers in $x_{t-m-1, t-m}$ across states up to date $t$, but excluding workers who return to employment between $t - m - 1$ and $t - m$. Thus, $x_{t-m-1, \tau}$ will be the time $\tau$ distribution of workers whose most recent exit from employment was to temporary-layoff unemployment between dates $t - m - 1$ and $\tau$. Denote $P_{\tau}$ to be the Markov transition matrix across $\{E, TL, JL, I\}$ at time $\tau$, mapping states at date $\tau - 1$ to $\tau$. Define $\tilde{P}_{\tau}^{i} = P_{\tau}^{i}$ for columns $i = TL, JL, I$, but $\tilde{P}_{\tau}^{i} = 0$ for column $i = E$. Then, given a distribution $x_{t-m-1, \tau-1}^{\prime}$ of workers at time $\tau - 1$ whose most recent exit from employment was to temporary-layoff unemployment between times $t - m - 1$ and $t - m$,

$$x_{t-m-1, \tau}^{\prime} = x_{t-m-1, \tau-1}^{\prime} \tilde{P}_{\tau}$$

gives the updated distribution of workers at time $\tau$. This updated distribution excludes workers who at any point return to employment between dates $\tau - 1$ and $\tau$; i.e., the $E$'th position of $x_{\tau-1} \tilde{P}_{\tau}$ equals zero. Thus, from initial condition $x_{t-m-1, t-m}$ and matrices $\{P_{\tau}\}_{\tau=t-m+1}^{t}$, we can calculate $x_{t-m-1, \tau}$ recursively for $\tau = t - m + 1, \ldots, t$.

We can calculate the number of workers in jobless unemployment at date $t$ whose most recent exit from employment was to temporary-layoff unemployment between dates $t - m - 1$ and $t - m$ as $e_{JL}^{\prime} x_{t-m-1, t}$, where $e_{JL}$ is a column vector with an entry of one in the $JL$'th place and zeros elsewhere. Then, the
number of workers in jobless unemployment at date $t$ whose most recent exit from employment was for temporary-layoff unemployment at some date in the last $\bar{T}$ periods is $\sum_{j=0}^{\bar{T}} e'_{jL} x_{t-j-1,t}$. 
Table A.1: Cyclical properties, gross worker flows

<table>
<thead>
<tr>
<th></th>
<th>$f_{ET}$</th>
<th>$f_{EP}$</th>
<th>$f_{TE}$</th>
<th>$f_{TP}$</th>
<th>$f_{PE}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>std($x$)/std($Y$)</td>
<td>11.264</td>
<td>4.962</td>
<td>6.609</td>
<td>10.084</td>
<td>7.126</td>
</tr>
<tr>
<td>Unemp. variance due to $x$</td>
<td>0.122</td>
<td>0.216</td>
<td>0.102</td>
<td>0.011</td>
<td>0.563</td>
</tr>
<tr>
<td>corr($x$, $Y$)</td>
<td>$-0.393$</td>
<td>$-0.674$</td>
<td>$0.599$</td>
<td>$-0.192$</td>
<td>$0.803$</td>
</tr>
</tbody>
</table>
Figure A.1: Labor market stocks and flows, 1980s recessions
Figure A.2: Labor market stocks and flows, 2008 recession
Figure A.3: Labor market stocks and flows, COVID-19 recession
Figure A.4: Total and temporary unemployment across sectors, 1980s recession
Figure A.5: Total and temporary unemployment across sectors, Great Recession
Figure A.6: Desired number of recalled workers versus available workers in temporary unemployment for firm ignoring recall-ability constraint