Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires

Supplementary Appendix

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A An approximate data-generating model

In this appendix, after describing the canonical Bils (1985) regression, we detail the approximate DGM introduced in Section 2 of the main text and use it to discuss inference of the canonical Bils regressions, where all new hires are pooled together, and our preferred regression specifications, where we distinguish between new hires from unemployment and employment. We also discuss inference with staggered contracting, and we detail how the DGM is consistent with the theoretical model of procyclical job upgrading that we develop in Section 3 of the main text. Throughout this appendix, we develop testable predictions that we employ in the main text of the paper that would allow us to reject the approximate DGM. As we discuss in the main text and mention here, however, the data are fully consistent with the approximate model.

A.1 Bils (1985) regressions

Bils (1985) estimates the following regression in first differences:

$$\Delta \log w_{it} = \Delta x_{it}' \cdot \pi_x + \pi_u \cdot \Delta u_t + \pi_n \cdot \mathbb{I}\{N_{it} = 1\} + \pi_{nu} \cdot \mathbb{I}\{N_{it} = 1\} \cdot \Delta u_t + e_{it}, \tag{1}$$

which regresses individual wages at time t on a cyclical indicator taken to be the unemployment rate and allows for a different cyclicality for continuing workers and new hires. The coefficient π_u gives the common wage semi-elasticity for new and continuing workers, while the coefficient π_{nu} measures the excess semi-elasticity for new hires. We note that in the original Bils specification, the indicator function is directly applied to first-differenced unemployment (as opposed to being first-differenced itself).

An alternative way to control for unobserved permanent heterogeneity is to estimate a regression in fixed effects. The fixed-effect version of (1) is

$$\Delta^m \log w_{it} = \Delta^m x_{it}' \cdot \pi_x + \pi_u \cdot \Delta^m u_t + \pi_n \cdot \mathbb{I}\{N_{it} = 1\} + \pi_{nu} \cdot \mathbb{I}\{N_{it} = 1\} \cdot \Delta^m u_t + e_{it}, \tag{2}$$

where Δ^m indicates mean-differences.

A.2 An approximate DGM with a cyclical composition effect

The DGM relates individual wages to individual factors, the state of the cycle (measured by unemployment) and to job match quality. There is no true excess wage flexibility for new hires. We then model match quality as procyclical for workers making job-to-job transitions (consistent with the structural model developed later in the paper). Accordingly, we can then show that because job-changers drive cyclicality among new hires, the Bils' regressions will generate upward biased estimates of new hire wage flexibility. Finally, we can use the DGM to motivate a test of whether new hires wages are excessively flexible. Under the assumption that match quality is on average acyclical for new hires coming from unemployment, we get an unbiased estimate of new hire wage flexibility by conditioning separately on this group of new hires.

Under the approximate DGM,

$$\log w_{it} = \psi_0 + x'_{it}\psi_x + \psi_u \cdot u_t + \alpha_i + \alpha_{it} + \varepsilon_{it}, \tag{3}$$

where ψ_0 is a constant, ψ_x describes the relation of wages to observable characteristics x_{it}, ψ_u gives the common wage semi-elasticity for new and continuing workers, α_i is a time-invariant person fixed effect, α_{it} is unobserved match quality for individual i at a given job at time t, and ε_{it} is an *iid* error term.

We assume average match quality $\bar{\alpha}_{it}$ evolves as follows: Let $\mathbb{I}\{EE_{it} = 1\}$ be an indicator that individual i is a new hire who has changed jobs and let $\mathbb{I}\{ENE_{it} = 1\}$ be an indicator that an individual is a new hire from unemployment. Then we assume that the average match quality $\bar{\alpha}_{it}$ evolves according to

$$\Delta\bar{\alpha}_{it} = \mathbb{I}\{EE_{it} = 1\} \cdot \left[\psi_n^{EE} + \psi_{nu}^{EE} \cdot \Delta u_t\right] + \mathbb{I}\{ENE_{it} = 1\} \cdot \psi_n^{ENE}. \tag{4}$$

Equation (4) describes a process for average match quality in which workers in continuing matches experience no change in match quality, workers hired from non-employment incur a level change in match quality independent of the cycle, and job changers incur a change in match quality that depends on a level component and the change in unemployment. The procyclical growth in match quality for job changers is consistent with the theoretical model of procyclical job upgrading that we develop in the later sections of the main text. As the unemployment rate falls, workers in existing jobs are more likely to move to better matches, and vice versus when the rate falls. By contrast, workers coming from unemployment have acyclical changes in match quality, as they are more likely to take the first job they find. We elaborate on this mechanism for job changers below (see Section A.6). In Section 4 of the main text, we verify that a similar condition also holds for job-changers in the full equilibrium model.

A.3 Inference with a first-difference estimator

To see how the typical regression of the literature may yield misleading evidence of new hire wage flexibility, we first take first differences of the DGM (3), integrate over changes in unobserved match quality $\Delta\alpha_{it}$ over new hires, and then combine with (4):

$$\begin{aligned}\Delta \log w_{it} &= \Delta x_{it}'\psi_x + \psi_u \cdot \Delta u_t \\ &\quad + \mathbb{I}\{EE_{it}=1\} \cdot \left[\psi_n^{EE} + \psi_{nu}^{EE} \cdot \Delta u_t\right] \\ &\quad + \mathbb{I}\{ENE_{it}=1\} \cdot \psi_n^{ENE} + \Delta\varepsilon_{it}. \end{aligned} \tag{5}$$

We then note that we can rewrite the Bils equation (1) in first differences as

$$\begin{aligned}\Delta \log w_{it} &= \Delta x_{it}'\pi_x + \pi_u \cdot \Delta u_t \\ &\quad + \mathbb{I}\{EE_{it}=1\} \cdot [\pi_n + \pi_{nu} \cdot \Delta u_t] \\ &\quad + \mathbb{I}\{ENE_{it}=1\} \cdot [\pi_n + \pi_{nu} \cdot \Delta u_t] + e_{it}, \end{aligned} \tag{6}$$

distinguishing between the two types of new hires. Under our DGM, the Bils regression is misspecified. In particular, it imposes that the wage semi-elasticities of job changers and new hires from unemployment are equal and given by π_{nu}. By contrast, the DGM implies that the elasticity for job changers is $\psi_{nu}^{EE} < 0$ and the elasticity for new hires from unemployment is zero. Accordingly, taking the Bils equation to the data will lead to an estimate $\widehat{\pi}_{nu} < 0$, but this will be due to the composition bias captured by ψ_{nu}^{EE}. Indeed, as we show next, $\widehat{\pi}_{nu} \propto \psi_{nu}^{EE}$.

In particular, from equations (1), (5) and (6), we have

$$\begin{aligned}\widehat{\pi}_{nu} &= \frac{\mathrm{Cov}^{M^X}\left(\Delta \log w_{it}, \mathbb{I}\{N_{it}=1\} \cdot \Delta u_t\right)}{\mathrm{Var}^{M^X}\left(\mathbb{I}\{N_{it}=1\} \cdot \Delta u_t\right)} \\ &= \psi_{nu}^{EE} \cdot \frac{\mathrm{Var}^{M^X}\left(\mathbb{I}\{EE_{it}=1\} \cdot \Delta u_t\right)}{\mathrm{Var}^{M^X}\left(\mathbb{I}\{N_{it}=1\} \cdot \Delta u_t\right)} \leq 0, \end{aligned}$$

where $X_{it} = \{\Delta x_{it}, \Delta u_t, I\{N_{it}=1\}\}$, $M^X = I - X\left(X'X\right)^{-1} X'$ is a linear operator that residualizes a vector with respect to X, and the superscript M^X denotes that we are computing covariances and variances with respect to the residualized arguments. Note, the literature takes the coefficient π_{nu} as a measure of the excess contractual flexibility of new hire wages; but under the approximate DGM, $\widehat{\pi}_{nu}$ is estimated to be non-zero solely through its proportionality to ψ_{nu}^{EE}.

We can then test whether the data is consistent with the simple DGM by estimating equation (6) allowing for separate interaction terms for job changers versus new hires from unemployment:

$$\begin{aligned}\Delta \log w_{it} &= \Delta x_{it}'\pi_x + \pi_u \cdot \Delta u_t \\ &+\mathbb{I}\left\{EE_{it}=1\right\} \cdot \left[\pi_n^{EE} + \pi_{nu}^{EE} \cdot \Delta u_t\right] \\ &+\mathbb{I}\left\{ENE_{it}=1\right\} \cdot \left[\pi_n^{ENE} + \pi_{nu}^{ENE} \cdot \Delta u_t\right] + e_{it},\end{aligned} \tag{7}$$

Under the null of our DGM,

$$\begin{aligned}\pi_{nu}^{EE} &= \psi_{nu}^{EE} \\ \pi_{nu}^{ENE} &= 0\end{aligned}$$

implying that (i) the excess wage cyclicality for job changers reflects composition bias and (ii) there is no excess wage cyclicality for new hires from unemployment. Of course, the data alone does not tell us the source of excess wage cyclicality for job changers. However, we can test the null that there should be no excess cyclicality for new hires from unemployment, i.e., whether $\pi_{nu}^{ENE} = 0$. Under our identifying assumptions π_{nu}^{ENE} provides a composition-free estimate of the excess wage flexibility of new hires (in contrast to π_{nu}^{EE}, which reflects cyclical composition). When we estimate equation (7) in Section 2.3 of the main text, we recover coefficients consistent with the null of $\pi_{nu}^{ENE} = 0$.

A.4 Inference with a fixed-effects estimator

The fixed-effects estimator shares the same crucial properties: we can apply it to a subset of our sample and obtain π_u and π_{nu}^{ENE} as consistent estimators for ψ_u and ψ_{nu}^{ENE}.

Similar to how we proceed in Section A.3, we first consider a fixed-effects estimator applied to the approximate DGM. We first take mean differences of equation (3), integrate over changes in unobserved match quality $\Delta^m \alpha_{it}$ and obtain

$$\Delta^m \log w_{it} = \psi_u \cdot \Delta^m u_t + \Delta^m \bar{\alpha}_{it} + \Delta^m \varepsilon_{it}, \tag{8}$$

where $\Delta^m \bar{\alpha}_{it}$ denotes average mean deviations in match quality. However, it requires more algebra to express (8) in a form that is readily comparable to the canonical regression equation, as we must generate expressions for mean deviations of unobserved match quality.

For ease of exposition, we assume in the following that individuals in the sample only make at most one job-transition; i.e., individuals will experience either one ENE transition, one EE transition, or will be continuing workers for the entire sample period. The length of the sample is denoted as T.

We first consider the mean-deviation of a worker who makes either a EE or a ENE

transition at period τ_i. Match quality for such newly hired worker evolves as

$$\alpha_{it} = \begin{cases} \alpha_{i,0} & \text{if } t < \tau_i \\ \alpha_{i,0} + \Delta\alpha_{\tau_i} & \text{if } t \geq \tau_i \end{cases}, \tag{9}$$

where $\Delta\alpha_{\tau_i}$ is the change in match quality experienced by the worker at time τ_i. When we average over T, we obtain the fixed-effect α_i^{FE}, where

$$\alpha_i^{FE} = \alpha_{i,0} + \frac{T - \tau_i + 1}{T} \cdot \Delta\alpha_{\tau_i}. \tag{10}$$

Then, the associated mean-deviations of the average match quality of an EE or ENE worker who moves to a new job at period τ_i is given by

$$\Delta^m \alpha_{it} = \begin{cases} -\left(\frac{T-\tau_i+1}{T}\right) \cdot \Delta\alpha_{\tau_i} & \text{if } t < \tau_i \\ \left(\frac{\tau_i - 1}{T}\right) \cdot \Delta\alpha_{\tau_i} & \text{if } t \geq \tau_i \end{cases}. \tag{11}$$

Integrating over unobserved match quality by types of job transition, we obtain

$$\Delta^m \bar{\alpha}_{it} = \begin{cases} -\left(\frac{T-\tau_i+1}{T}\right) \cdot \Delta\bar{\alpha}_{\tau_i} & \text{if } t < \tau_i \\ \left(\frac{\tau_i - 1}{T}\right) \cdot \Delta\bar{\alpha}_{\tau_i} & \text{if } t \geq \tau_i \end{cases} \tag{12}$$

where, analogously to equation (4), $\Delta\bar{\alpha}_{\tau_i}$ is given by

$$\Delta\bar{\alpha}_{\tau_i} = \mathbb{I}\left\{\text{ever } EE_i = 1\right\} \cdot \left[\psi_n^{EE} + \psi_{nu}^{EE} \cdot \Delta u_{\tau_i}\right] + \mathbb{I}\left\{\text{ever } ENE_i = 1\right\} \cdot \psi_n^{ENE}, \tag{13}$$

and where $\mathbb{I}\{\text{ever } EE_i = 1\}$ is an indicator variable equal to one if the worker makes one EE transition over the sample period and zero otherwise, $\mathbb{I}\{\text{ever } ENE_i = 1\}$ is an indicator variable equal to one if a worker makes an ENE transition over the sample period and zero otherwise, and τ_i is the date of the job transition of a worker who makes either an EE or ENE transition.

Finally, we can rewrite (12) as

$$\Delta^m \bar{\alpha}_{it} = \left[\mathbb{I}\left\{t < \tau_i\right\} \cdot \left(-\frac{T - \tau_i + 1}{T}\right) + \mathbb{I}\left\{t \geq \tau_i\right\} \cdot \left(\frac{\tau_i - 1}{T}\right)\right] \Delta\bar{\alpha}_{\tau_i}. \tag{14}$$

Equations (8), (13), and (14) describe the approximate DGM in fixed effects.

We next rewrite the analogue of our baseline regression equation for wage cyclicality (7) in fixed effects,

$$\begin{aligned}\Delta^m \log w_{it} \quad &= \quad \Delta^m x_{it}' \pi_x + \pi_u \cdot \Delta^m u_t \\ &\quad + \mathbb{I}\{EE_{it} = 1\} \cdot \left[\pi_n^{EE} + \pi_{nu}^{EE} \cdot \Delta^m u_t\right] \\ &\quad + \mathbb{I}\{ENE_{it} = 1\} \cdot \left[\pi_n^{ENE} + \pi_{nu}^{ENE} \cdot \Delta^m u_t\right] + e_{it}, \end{aligned} \tag{15}$$

While it was readily apparent that the preferred regression equation in first-differences would allow consistent estimates of the parameters of the DGM, it is perhaps less clear that this is true for our fixed effects estimator. Here, we show that π_{nu}^{ENE} is a consistent estimator of ψ_{nu}^{ENE}, π_u is a consistent estimator for ψ_u when we estimate equation (15) over a subset of the full sample, but that π_{nu}^{EE} is a biased estimator for ψ_{nu}^{EE}.

A.4.1 Consistent estimator for ψ_{nu}^{ENE} in fixed-effects

Let $X_{it} = \{\Delta^m x_{it}, \Delta^m u_t, \mathbb{I}\{EE_{it} = 1\}, \mathbb{I}\{EE_{it} = 1\} \cdot \Delta^m u_t, \mathbb{I}\{ENE_{it} = 1\}\}$ and let M^X denote the linear operator that residualizes a vector with respect to X. Then, by the Frisch-Waugh theorem,

$$\begin{aligned}\widehat{\pi}_{nu}^{ENE} &= \frac{\mathrm{Cov}^X\left(\Delta^m \log w_{it}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\right)}{\mathrm{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)} \\ &= \frac{\mathrm{Cov}^X\left(\Delta x_{it}' \psi_x + \psi_u \cdot \Delta^m u_t + \Delta^m \bar{\alpha}_{it} + \Delta^m \varepsilon_{it}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\right)}{\mathrm{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)} \\ &= 0 + \frac{\mathrm{Cov}^X\left(\Delta^m \bar{\alpha}_{it}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\right)}{\mathrm{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)} \\ &= \frac{\mathrm{Cov}^X\left(\mathbb{I}\{\text{ever } ENE_i = 1\} \cdot \mathbb{I}\{t < \tau_i\}\left(-\frac{T - \tau_i + 1}{T}\right) \cdot \psi_n^{ENE}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\right)}{\mathrm{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)} \\ &+ \frac{\mathrm{Cov}^X\left(\mathbb{I}\{\text{ever } ENE_i = 1\} \cdot \mathbb{I}\{t \geq \tau_i\} \cdot \left(\frac{\tau_i - 1}{T}\right) \cdot \psi_n^{ENE}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\right)}{\mathrm{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)} \\ &= 0 \end{aligned} \tag{16}$$

where first equality follows from equation (15) and residualization, the second equality from equation (8), the third equality from residualization, the fourth equality from equations (13) and (14), and the final equality follows from the zero covariance of τ_i and $\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t$.[1] The estimates we obtain in Section 2.3 of the main text are consistent with the null of $\pi_{nu}^{ENE} = 0$.

[1] Note, $\mathrm{Cov}(\tau_i, \Delta^m u_t) = 0$ obtains from the random initial condition for u_t.

A.4.2 Consistent estimator for ψ_u in fixed-effects

First, note that we cannot obtain consistent estimates of ψ_u from a full sample that includes workers who ever make an EE transition. As shown in equation (13), the mean deviations of wages for ever-EE workers includes a term that is proportional to the change in the unemployment rate of the period that they make a job-transition. As the fixed-effects regression does not include this as an explanatory variable, estimates of ψ_u will suffer from omitted variable bias. We can, however, derive consistent estimates of ψ_u if we consider a restricted sample that omits ever-EE workers. Let $X_{it} = \{\Delta^m x_{it}, \mathbb{I}\{ENE_{it} = 1\}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\}$ and let M^X denote the linear operator that residualizes a vector with respect to X.

Then,

$$\begin{aligned}
\widehat{\pi}_u &= \frac{\text{Cov}^X\left(\Delta^m \log w_{it}, \Delta^m u_t\right)}{\text{Var}^X(\Delta^m u_t)} \\
&= \frac{\text{Cov}^X\left(\Delta x_{it}'\psi_x + \psi_u \cdot \Delta^m u_t + \Delta^m \bar{\alpha}_{it} + \Delta^m \varepsilon_{it}, \Delta^m u_t\right)}{\text{Var}^X(\Delta^m u_t)} \\
&= \psi_u + \frac{\text{Cov}^X\left(\Delta^m \bar{\alpha}_{it}, \Delta^m u_t\right)}{\text{Var}^X(\Delta^m u_t)} \\
&= \psi_u + \frac{\text{Cov}^X\left(\mathbb{I}\{\text{ever } ENE_i = 1\} \cdot \mathbb{I}\{t < \tau_i\} \cdot \left(-\frac{T-\tau_i+1}{T}\right) \cdot \psi_n^{ENE}, \Delta^m u_t\right)}{\text{Var}^X(\Delta^m u_t)} \\
&+ \frac{\text{Cov}^X\left(\mathbb{I}\{\text{ever } ENE_i = 1\} \cdot \mathbb{I}\{t \geq \tau_i\} \cdot \left(\frac{\tau_i - 1}{T}\right) \cdot \psi_n^{ENE}, \Delta^m u_t\right)}{\text{Var}^X(\Delta^m u_t)} \\
&= \psi_u
\end{aligned} \tag{17}$$

where first equality follows from equation (15) and residualization, the second equality from equation (8), the fourth equality from equations (13) and (14), and the final equality follows from the zero covariance of τ_i and $\Delta^m u_t$.

A.4.3 Biased estimator for ψ_{nu}^{EE} in fixed-effects

Let $X_{it} = \{\Delta^m x_{it}, \Delta^m u_t, \mathbb{I}\{EE_{it} = 1\}, \mathbb{I}\{ENE_{it} = 1\}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\}$ and let M^X denote the linear operator that residualizes a vector with respect to X. Without loss of generality, assume that $\psi_n^{ENE} = 0$ and $\psi_n^{EE} = 0$. Then,

$$\begin{aligned}
\widehat{\pi}_{nu}^{EE} &= \frac{\text{Cov}^X(\Delta^m \log w_{it}, \mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)} \qquad (18)\\
&= \frac{\text{Cov}^X(\Delta x_{it}'\psi_x + \psi_u \cdot \Delta^m u_t + \Delta^m \bar{\alpha}_{it} + \Delta^m \varepsilon_{it}, \mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}\\
&= \frac{\text{Cov}^X(\Delta^m \bar{\alpha}_{it}, \mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}\\
&= \psi_{nu}^{EE} \cdot \frac{\text{Cov}^X((\frac{\tau_i-1}{T}) \cdot \mathbb{I}\{EE_{it}=1\} \cdot \Delta u_{\tau_i}, \mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}\\
&+ \frac{\text{Cov}^X(\mathbb{I}\{t<\tau_i\} \cdot \left(-\frac{T-\tau_i+1}{T}\right) \cdot \mathbb{I}\{\text{ever } EE_i=1\} \cdot \psi_{nu}^{EE} \cdot \Delta u_{\tau_i}, \mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}\\
&+ \frac{\text{Cov}^X(\mathbb{I}\{t>\tau_i\} \cdot (\frac{\tau_i-1}{T}) \cdot \mathbb{I}\{\text{ever } EE_i=1\} \cdot \psi_{nu}^{EE} \cdot \Delta u_{\tau_i}, \mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{EE_{it}=1\} \cdot \Delta^m u_t)}\\
&\neq \psi_{nu}^{EE}
\end{aligned}$$

where the first equality follows from equation (15) and residualization, the second equality from (8), and the fourth equality follows from equations (13) and (14). At the fourth equality, however, we see two additional terms on the second and third line where a component of demeaned average match quality is correlated with the demeaned unemployment rate in the periods before and after the period a worker is an EE new hire, from equations (11) and (12). Even if these terms summed to zero, $(\tau_i - 1)/T < 1$ and $\text{Cov}(\Delta u_{\tau_i}, \Delta^m u_t) \neq \text{Var}(\Delta^m u_t)$, so we would obtain a biased estimate.

A.5 Robustness to staggered contracting

The structural model we develop in the paper allows for staggered wage contracting, where the likelihood of renegotiation obeys a Poisson process. To introduce staggered contracting, we slightly modify the approximate DGM:

$$\log w_{it} = \log w_{it}^f + x_{it}'\psi_x + \alpha_i + \alpha_{it} + \varepsilon_{it} \qquad (19)$$

where w_{it}^f is the prevailing wage at a firm and is recontracted with probability $(1-\lambda)$:

$$\log w_{it}^f = \begin{cases} \psi_0 + \psi_u \cdot u_t & \text{w/ prob. } (1-\lambda) \\ \log w_{it-1}^f & \text{w/ prob. } \lambda \end{cases}$$

With a bit of algebra, one can then develop the analogue of equation (7), our baseline relation for wage cyclicality which allows for separate interactions for job changers and new hires from unemployment.

$$\begin{aligned}\Delta \log w_{it} = \Delta x_{it}' \pi_x + \pi_u \cdot (1-\lambda) \sum_{\tau=0}^{\infty} \lambda^{\tau} \Delta u_{t-\tau} \\ + \mathbb{I}\{EE_{it} = 1\} \cdot \left[\pi_n^{EE} + \pi_{nu}^{EE} \cdot \Delta u_t\right] \\ + \mathbb{I}\{ENE_{it} = 1\} \cdot \left[\pi_n^{ENE} + \pi_{nu}^{ENE} \cdot \Delta u_t\right] + e_{it}, \end{aligned} \tag{20}$$

Note the difference with (7) is that the cyclical indicator is now a distributed lag of current and past unemployment growth rates as opposed to just the current one. In the final section of this appendix, we show that all our results are robust to this alternative formulation; that is, the estimate of π_{nu}^{ENE} is still zero, implying no excess wage flexibility for new hires. The estimate of π_{nu}^{EE} remains negative, but this reflects cyclical composition bias under our identifying assumptions.

A.6 Average match quality with constant wage premium and cyclical shares

We detail here how the DGM is consistent with the theoretical model of procyclical job upgrading that we develop in the paper. In particular, we show we can write the procyclical growth in average match quality for job changers in equation (4) as generated by a constant wage premium across types of jobs and cyclical changes in the probability of moving across job types, as predicted by our full equilibrium model.

Suppose that there are two types of match quality, good and bad. Bad matches are paid a fraction ϕ of good matches. We observe whether a worker makes a direct job-to-job transition or one with an intervening spell of nonemployment, but nothing else about the job transition. The probability of making a bad-to-good (or good-to-bad) transition depends on the cycle for job-changers, but not for new hires from unemployment.

Then, conditional on a EE transition,

$$\Delta \alpha_{it} = \begin{cases} -\log \phi & \text{w/ prob. } \delta_{BG,t} = \eta_{BG,0} + \eta_{BG,u} \cdot \Delta u_t \\ \log \phi & \text{w/ prob. } \delta_{GB,t} = \eta_{GB,0} + \eta_{GB,u} \cdot \Delta u_t \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

where $\delta_{BG,t}$ indicates the fraction of job-changers who make a bad-to-good transition and where

$$\delta_{BG,t} + \delta_{GB,t} + \delta_{BB,t} + \delta_{GG,t} = 1. \tag{22}$$

Conditional on a ENE transition,

$$\Delta\alpha_{it} = \begin{cases} -\log\phi & \text{w/ prob. } \delta_{BNG,t} = \eta_{BNG,0} \\ \log\phi & \text{w/ prob. } \delta_{GNB,t} = \eta_{GNB,0} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

where $\delta_{BNG,t}$ is the fraction of new hires from unemployment who make a bad-to-good transition (with an intervening spell of unemployment) and where

$$\delta_{BNG,t} + \delta_{GNB,t} + \delta_{BNB,t} + \delta_{GNG,t} = 1. \tag{24}$$

If we integrate over the probability of different types of match transitions, we obtain

$$\begin{aligned} \Delta\bar{\alpha}_{it} = \,& \mathbb{I}\{EE_{it} = 1\} \cdot (-\log\phi) \cdot \left(\eta_{BG,0} - \eta_{GB,0}\right) \\ &+ \mathbb{I}\{EE_{it} = 1\} \cdot (-\log\phi) \cdot \left(\eta_{BG,u} - \eta_{GB,u}\right) \cdot \Delta u_t \\ &+ \mathbb{I}\{ENE_{it} = 1\} \cdot (-\log\phi) \cdot \left(\eta_{BNG,0} - \eta_{GNB,0}\right) \end{aligned} \tag{25}$$

This is nested in (4), where

$$\begin{aligned} \psi_n^{EE} &= (-\log\phi) \cdot \left(\eta_{BG,0} - \eta_{GB,0}\right) \\ \psi_{nu}^{EE} &= (-\log\phi) \cdot \left(\eta_{BG,u} - \eta_{GB,u}\right) \\ \psi_n^{ENE} &= (-\log\phi) \cdot \left(\eta_{BNG,0} - \eta_{GNB,0}\right) \end{aligned}$$

A.7 Robustness to alternative approximate DGMs

Here, we consider three deviations to our preferred regression specification that are motivated by three changes to the baseline approximate DGM. As in the text, we consider the canonical regression, where all new hires are treated the same, and our preferred specification, where new hires from employment and unemployment are permitted separate wage cyclicalities. Results are given in Tables A.1 and A.2. First, we consider the case in which the average change in match quality for workers making job-to-job transitions is proportional to the level unemployment rate rather than the change in the unemployment rate. Hence, whereas the common cyclical indicator is expressed in first differences and mean deviations, the interaction term is expressed in the level HP-filtered unemployment rate. Second, we consider the case of staggered contracting, where the relevant cyclical indicator is the distributed lag of unemployment, as in Section A.5, where we set $\lambda = 11/12$. The third case we consider is a combination of the first two. As seen in Table A.1, we obtain a new hire effect à la Bils (1985) across all three specifications. We confirm in Table A.2, however, that the effect is entirely driven by new hires from employment, and there is no evidence for excess wage cyclicality for new hires from non-employment.

Table A.1: Alternative cyclical indicators: canonical regression

	First differences			Fixed-effects		
UR	−0.520***	−1.090***	−1.125***	−0.192***	−0.965***	−0.995***
	(0.0992)	(0.1377)	(0.1401)	(0.0602)	(0.0699)	(0.0700)
UR · $\mathbb{I}$(new)	−0.280**	−0.896*	−0.211*	−0.183***	−1.196***	−0.194***
	(0.1224)	(0.4583)	(0.1235)	(0.0612)	(0.3246)	(0.0612)
$\mathbb{I}$(new)	0.292**	0.010**	0.222*	0.172***	−0.011***	0.183***
	(0.1236)	(0.0040)	(0.1247)	(0.0617)	(0.0017)	(0.0617)
Change to cyclical indicators?	Level Inter'n	Distr. lag	Level & Distr. lag	Level Inter'n	Distr. lag	Level & Distr. lag
No. observations	321,396	321,396	321,396	378,661	378,661	378,661
No. individuals	57,265	57,265	57,265	57,265	57,265	57,265
No. new hires	14,674	321,396	321,396	18,096	18,096	18,096

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.

Table A.2: Alternative cyclical indicators: preferred specification

A.2A. First differences

	Level interactions		Distributed lag		Level interactions & distributed lag	
UR	−0.482*** (0.0993)	−0.487*** (0.0989)	−1.060*** (0.1380)	−1.051*** (0.1380)	−1.088*** (0.1404)	−1.091*** (0.1400)
UR · $\mathbb{I}(EE)$	−0.334** (0.1467)	−0.355** (0.1454)	−1.649** (0.6793)	−1.452** (0.6209)	−0.273* (0.1475)	−0.294** (0.1463)
UR · $\mathbb{I}(ENE)$	−0.203 (0.2245)	−0.090 (0.2460)	−0.161 (0.6597)	−0.257 (0.7312)	−0.119 (0.2252)	0.000 (0.2465)
Unemp. spell for ENE	0+	1+	0+	1+	0+	1+
$P(\pi_{nu}^{EE} = \pi_{nu}^{ENE})$	0.627	0.356	0.112	0.209	0.564	0.305

A.2B. Fixed effects

	Interaction of levels		Distributed lag		Interaction of levels & distributed lag	
UR	−0.169*** (0.0608)	−0.170*** (0.0608)	−0.957*** (0.0704)	−0.957*** (0.0704)	−0.972*** (0.0705)	−0.972*** (0.0705)
UR · $\mathbb{I}(EE)$	−0.304*** (0.0822)	−0.287*** (0.0805)	−1.590*** (0.5028)	−1.547*** (0.4726)	−0.318*** (0.0823)	−0.300*** (0.0806)
UR · $\mathbb{I}(ENE)$	0.081 (0.1059)	0.173 (0.1106)	0.059 (0.5388)	0.438 (0.5934)	0.074 (0.1057)	0.166 (0.1103)
Unemp. spell for ENE	0+	1+	0+	1+	0+	1+
$P(\pi_{nu}^{EE} = \pi_{nu}^{ENE})$	0.004	0.001	0.023	0.008	0.003	0.001

Robust standard errors in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.

B Empirical appendix

B.1 More on robustness of empirical results

In this section, we discuss several issues related to the robustness of our empirical findings.

B.1.1 Non-employment versus unemployment

We interpret our findings for the wage cyclicality of workers from non-employment to be relevant to understanding the wage cyclicality of workers from unemployment. The SIPP offers monthly data on search activity, but many workers do not report active search for each month of a given spell. Hence, there is no straightforward criteria by which to classify a complete non-employment spell as one of "unemployment", which would be necessary to refine our measure of ENE transitions to one of EUE transitions.

We have explored one possibility, however, which is to classify a new hire as a new hire from unemployment (EUE) if the worker reports searching for at least a single month of her non-employment spell. Roughly 72% of new hires from non-employment in our sample report searching for at least one month of non-employment. Of those workers, they search for around 86% of the total duration of their non-employment spell. From this measure, we can define three types of new hires: EE, EUE, and EOE (new hires from OLF). We estimate a variation of our baseline specification with three interaction terms for each type of new hire. The results are reported in Table B.1. Our findings are virtually unchanged. If anything, the point estimates suggest that the wages of EUE workers are less cyclical than the wages of ENE workers, but this difference is not statistically significant.

B.1.2 More findings on cyclical selection from non-employment

In Section 2.4 of the main paper, we discuss the robustness of our findings when isolate groups of ENE workers who are less likely to be subject to some form of cyclical composition bias. In Tables 4 and 5 of the main text, we isolate workers with shorter durations of non-employment, where we alternatively do not control or control for occupation switchers. Here, we do the same, but for with industry switchers. Results are given in Tables B.2 and B.3 and entirely consistent with those presented in Tables 4 and 5.

B.2 The Survey of Income and Program Participation

Here, we discuss issues related to the Survey of Income and Program Participation as they pertain to our analysis.

B.2.1 Overview

The SIPP is administered by the U.S. Census Bureau and is designed to track a nationally representative sample of U.S. households. The SIPP is organized by panel years, where each panel year introduces a new sample of households. From 1990 to 1993, the Census Bureau would introduce a new panel on an annual basis, where each panel is administered for a period of 32 to 40 months. Hence for certain years in the early 1990s, data is available from multiple panels, each consisting from around 15,000 to 24,000 households. Starting in 1996, the Census changed the structure of the survey to follow contiguous panels. Since the redesign, new panels have been introduced in 1996, 2001, 2004, and 2008. For each of these panels, the Census has followed a larger sample of households (e.g. 40,188 in 1996) over a longer period. There are two inter-panel periods for which we have no coverage: April 2000 to February 2001, and February 2008 to August 2008. We use the SIPP sample weights for all of our analysis. For the periods where there is panel overlap, we adjust the weights according to the procedure recommended by Census (e.g., see SIPP User's Guide, 2001, pages 8-9).

Survey respondents are interviewed every four months on activity since the previous interview, a period referred to as a *wave*. However, some information (including for example, employment) is available at different frequencies within a wave. For example, the SIPP provides weekly measures of employment status, monthly measures of earnings, and job identifiers are constant for the entire period of the wave. As described in the main text, we combine monthly earnings records specific to each job to discern the pattern of job flows and sources of earnings over the wave.

The SIPP has several advantages relative to other commonly used panel data sources such as the PSID or the NLSY. Relative to the PSID, the SIPP follows a larger number of households, is nationally representative, and has more frequent observations. For the purposes of this paper, the PSID also suffers the disadvantage that it is difficult to identify wage earnings with a particular job in years where multiple jobs are held. Relative to the NLSY, the SIPP follows a larger number of households, but more importantly, multiple cohorts. Relative to both surveys, the SIPP suffers the disadvantage that it follows any particular individual for a shorter overall duration. But as mentioned before, the SIPP collects rich retrospective information that gets around problems of left-censoring: in particular, we observe start dates for jobs held during the first wave but started prior to the first interview (including the 1990 to 1993 panels). We then use our earnings-based measures of job transitions to determine the following sequence of jobs spells for the rest of the sample.[2]

[2] For each wave, the survey contains fields for up to two jobs. The survey maintains longitudinally consistent job IDs for each individual and tracks certain job-specific characteristics at a monthly frequency,

One frequently noted problem with the SIPP is the so-called "seam effect," described by the Census Bureau as follows:

This effect results from the respondent tendency to project current circumstances back onto each of the 4 prior months that constitute the SIPP reference period. When that happens, any changes in respondent circumstances that occurred during that 4-month period appear to have happened in the first month of the reference period. A disproportionate number of changes appear to occur between the fourth month of one wave and the first month of the following wave, which is the "seam" between the two wave–hence the terminology. (SIPP User's Guide 2001, pages 1-7)

For our purposes, such an effect could potentially generate problems, for example, with aligning job-changes with changes in the unemployment rate. This sort of measurement error has the potential to bias coefficient estimates towards zero. Having experimented with different measures of the unemployment rate that should not lead to such bias, e.g. average unemployment rate over the wave, we believe the quantitative impact of such measurement error is negligible. While a downward bias would make it harder to reject the null hypothesis of no excess cyclicality of ENE wages, we recover both small negative and positive coefficient estimates for ENE workers, so we are not suspicious that such a bias could be driving our results. Moreover, given there is no ex-ante reason to believe that such measurement error should differentially effect ENE versus EE workers, the bias would not make it easier to reject that excess wage cyclicality of EE and ENE workers are the same.

A second problem from the seam effect is discussed in Gottschalk (2005), which is the presence of measurement error for the amount of earnings in non-interview months of a wave. Thus, we follow Gottschalk (2005) and others in only using the fourth month wage in our analysis.

Finally, we note that some sort of "seam bias" is present for any retrospective data. In this sense, the SIPP provides an almost-ideal survey instrument, as it offers a representative sample for multiple cohorts with a rich set of variables, but also allows the researcher to follow workers over several years with relatively high frequency interviews, limiting the duration over which the seam bias can influence survey responses.

including earnings. We follow the procedure detailed by Stinson (2003) to correct inconsistent job identification variables for the 1990 to 1993 panels. We use monthly earnings data within waves to determine at which job the individual is working and for what months the individual is working at each potential job. From these data, we determine within a wave whether an individual made a job transition; and whether the job transition was characterized by an intervening period of non-employment.

B.2.2 Variables and sample selection

Following Bils (1985), we only consider males between the ages of 20 and 60. We drop observations for individuals who are disabled, self-employed, serving in the armed forces, or enrolled in school full-time. We use the monthly employment status recode variable to identify and drop observations where an individual reports not working for the entire month. We drop observations where an individual works less than 10 or greater than 100 hours a week. We also drop observations where the wage is top-coded or below the minimum wage. All observations are associated with a job-specific wage. As such, we drop observations where a worker is working at multiple jobs. Such observation may either reflect a job-to-job transition or multiple job-holding; but in either case, it is difficult to determine which observation should be included in the estimation.

We use hourly wages as our measure of earnings. In some instances, SIPP includes hourly wages and total monthly earnings. In cases where the hourly wage is directly available, we use that as our measure of wages. In cases where the hourly wage is not available, we construct a measure of implied hourly wages from monthly earnings divided by the product of weeks worked and hours worked per week. For the 1990 to 1993 waves, all of these variables are job-specific. Starting with the 1996 panel, the measure for weeks worked is no longer job-specific. We instead construct a measure of weeks worked from weeks with job minus weeks absent from work. Note that the implied hourly wage measure is subject to greater measurement error at the beginning of a job, when an individual does not necessarily spend a full month working at a job. In such cases, we use the second observation as the "new hire" wage. There is no considerable change for the fixed effects regression if we do not apply this correction, but many of the coefficients are not statistically significant for the first-differences regression, including for new hires from employment. We deflate wages using a four-month average of the PCE. Covariates include four indicators for educational attainment, separate indicators for union coverage and marital status, a quadratic in job tenure, and a time trend. We use combined weights across panels, applying the method recommended by the SIPP User's Guide (2001). We use monthly prime-aged male unemployment.

B.2.3 Identifying recalls

The SIPP maintains job-specific longitudinally consistent employment information over waves for which an individual reports non-zero employment. For such case, the SIPP maintains the same job identifier for a given job, allowing users to distinguish new jobs from "recalls" (to adopt the terminology of Fujita and Moscarini (2017). Table B.4 gives an example employment history of an individual who works at a job, spends four months in non-employment, but returns to the same job. The SIPP correctly records that the individual returned to the job that she left.

But starting in 1996, the SIPP resets employment records for individuals who are without employment for an entire wave. If individuals return to a previously held job after spending an entire wave in non-employment, the SIPP will incorrectly record the individual as starting a new job. Hence, a single job can be given multiple job identifiers. Table B.5 gives a sample employment history of an individual who works at a job, spends an entire wave out of work, and then returns to the same job. As in the previous example, the individual spends four months not working; but because those four months happen to fall over the entirety of a wave, the job is given a new identifier when the individual returns to work. For such individuals, we could mistakenly label a recall to be a transition across separate jobs.

We exploit an additional source of information recorded by the SIPP to identify potential recalls. Every time that a distinct job identifier is associated with an individual, the survey also adds a start date. This is indicated by the box around "start date" in the third row of Table B.5. When we observe a start date that falls before the date that the SIPP purges job identifiers, we have a good indication that the "new job" is in fact a recall.

To what extent do respondents report the date that they began the job, inclusive of employment gaps, versus the date that they last began a contiguous employment spell? We note that the survey question recording start dates is explicitly designed to identify the start date to be the former of the two, as it is designed to distinguish jobs that began within the wave from jobs that began before the wave.

For example, in the 1996 panel, respondents are asked "Did [FIRST AND LAST NAME] begin [HIS HER] employment with [NAME OF EMPLOYER] at some time between [MONTH1] 1st and today?" (variable STRTJB). If individuals respond in the affirmative, they are asked about the month and day within the wave that the job began (STRTREFP). Otherwise, they are asked to give their "BEST estimate" of the year, month, and date that the job began (variables STRTMONJB, STRTJYR, STRTJMTH).[3]

To identify potential recalls, we apply the following criterion: for individuals with an incomplete employment record – e.g. respondents who have spent a complete wave in non-employment – we consider any job with a start date prior to the period of non-employment (the date at which the SIPP purges internal employment records) as a potential recall, and we do not count the individual as a new hire.

We illustrate our criterion in Tables B.6 and B.7. In Table B.6, we observe an individual work a wave at Job A, spend an entire wave in non-employment, and then start work at

[3] See the 1996 Panel Wave 02 Questionnaire at http://www.census.gov/content/dam/Census/programs-surveys/sipp/questionnaires/1996/SIPP%201996%20Panel%20Wave%2002%20-%20Core%20Questionnaire.pdf

Job B in wave 3. The start date of Job B is before the "gap date", and hence, it is more likely that Job B is the same as Job A. Hence, we do not consider the individual as a new hire at Job B. In Table B.7, we similarly observe an individual work at Job A, spend a wave in non-employment, and then work at Job B; however, the start date for job B in this instance is after the gap date, and hence, we consider the worker to be a new hire in wave 3.

We apply the gap date criterion with two small additions: first, for a subset of job dissolutions, workers report the cause of the dissolution. If the worker reports that he left the pre-gap job to take another job, we do preclude the possibility that the post-gap job is a recall to the first job. Second, if the start date at a post gap job is missing or statistically imputed, we identify the job as a potential recall and do not count the worker as a new hire.

Table B.1: EE, EUE, & ENE

	FD		FE	
	(1)	(2)	(3)	(4)
UR	−0.424***	−0.420***	−0.144**	−0.145**
	(0.0966)	(0.0966)	(0.0609)	(0.0609)
UR · $\mathbb{I}(EE)$	−1.866***	−1.666***	−1.974***	−1.933***
	(0.6794)	(0.6218)	(0.5027)	(0.4724)
UR · $\mathbb{I}(EUE)$	−0.317	−0.479	−0.022	0.236
	(0.7308)	(0.7949)	(0.5683)	(0.6099)
UR · $\mathbb{I}(EOE)$	−0.950	−0.801	−1.348	−0.557
	(1.6688)	(2.0143)	(1.0456)	(1.2213)
$\mathbb{I}(EE)$	0.045***	0.038***	0.004*	0.001
	(0.0048)	(0.0046)	(0.0023)	(0.0022)
$\mathbb{I}(EUE)$	−0.057***	−0.075***	−0.034***	−0.039***
	(0.0071)	(0.0081)	(0.0032)	(0.0037)
$\mathbb{I}(EOE)$	−0.010	−0.015	−0.020***	−0.013*
	(0.0152)	(0.0176)	(0.0064)	(0.0074)
$P(\pi_{nu}^{EE} = \pi_{nu}^{EUE})$	0.117	0.235	0.009	0.004
Unemp. spell for ENE	0+	1+	0+	1+
No. observations	318,763	318,763	375,642	375,642

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.

Table B.2: ENE by unemployment duration, controls for industry switchers, first differences

	$\leq$ 9 months		$\leq$ 8 months		$\leq$ 7 months		$\leq$ 6 months		$\leq$ 5 months	
UR	-0.409^{***}	-0.408^{***}	-0.408^{***}	-0.407^{***}	-0.407^{***}	-0.406^{***}	-0.406^{***}	-0.405^{***}	-0.407^{***}	-0.407^{***}
	(0.0966)	(0.0966)	(0.0966)	(0.0966)	(0.0966)	(0.0966)	(0.0966)	(0.0966)	(0.0966)	(0.0966)
UR $\cdot$ $\mathbb{I}(EE)$	-1.855^{***}	-2.225^{**}	-1.853^{***}	-2.224^{**}	-1.853^{***}	-2.223^{**}	-1.852^{***}	-2.222^{**}	-1.854^{***}	-2.223^{**}
	(0.6795)	(0.8802)	(0.6796)	(0.8801)	(0.6796)	(0.8801)	(0.6796)	(0.8801)	(0.6796)	(0.8801)
UR $\cdot$ $\mathbb{I}(ENE)$	−0.460	0.238	−0.698	0.207	−0.896	0.445	−0.619	0.265	−0.673	0.758
	(0.7499)	(1.0419)	(0.7546)	(1.0494)	(0.7085)	(0.9709)	(0.7380)	(1.0250)	(0.7595)	(0.9825)
UR $\cdot$ $\mathbb{I}(LTU)$	−1.438	0.449	−1.078	0.219	−0.600	−1.084	−0.970	0.049	−0.709	−0.977
	(1.2929)	(1.6967)	(1.2556)	(1.6946)	(1.4762)	(2.6394)	(1.3015)	(2.1089)	(1.1779)	(2.0159)
UR $\cdot$ $\mathbb{I}(EE$ & switcher)	—	0.717	—	0.718	—	0.717	—	0.717	—	0.715
		(1.3352)		(1.3353)		(1.3353)		(1.3353)		(1.3353)
UR $\cdot$ $\mathbb{I}(ENE$ & switcher))	—	−1.298	—	−1.640	—	-2.392^{*}	—	−1.616	—	-2.573^{*}
		(1.4644)		(1.4772)		(1.3809)		(1.4435)		(1.4636)
UR $\cdot$ $\mathbb{I}(LTU)$ & switcher)	—	−2.616	—	−1.807	—	0.535	—	−1.550	—	0.214
		(2.3023)		(2.2727)		(3.1624)		(2.6388)		(2.4669)
Ind. controls	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
$P(\pi_{nu}^{EE} = \pi_{nu}^{ENE})$	0.163	0.070	0.250	0.075	0.324	0.041	0.214	0.065	0.243	0.023
No. observations	375,642	375,641	375,642	375,641	375,642	375,641	375,642	375,641	375,642	375,641

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.

Table B.3: ENE by unemployment duration, controls for industry switchers, fixed-effects

	$\leq$ 9 months		$\leq$ 8 months		$\leq$ 7 months		$\leq$ 6 months		$\leq$ 5 months	
UR	-0.145^{**}	-0.145^{**}	-0.144^{**}	-0.145^{**}	-0.144^{**}	-0.144^{**}	-0.144^{**}	-0.144^{**}	-0.145^{**}	-0.144^{**}
	(0.0609)	(0.0609)	(0.0609)	(0.0609)	(0.0609)	(0.0609)	(0.0609)	(0.0609)	(0.0609)	(0.0609)
UR $\cdot\ \mathbb{I}(EE)$	-1.971^{***}	-2.859^{***}	-1.971^{***}	-2.859^{***}	-1.971^{***}	-2.859^{***}	-1.971^{***}	-2.859^{***}	-1.971^{***}	-2.859^{***}
	(0.5027)	(0.6025)	(0.5027)	(0.6025)	(0.5027)	(0.6025)	(0.5027)	(0.6025)	(0.5027)	(0.6025)
UR $\cdot\ \mathbb{I}(ENE)$	-0.246	1.215^{*}	-0.333	1.155	-0.387	1.200	-0.387	1.243	-0.173	1.449^{*}
	(0.5816)	(0.7285)	(0.5882)	(0.7344)	(0.6040)	(0.7446)	(0.6245)	(0.7643)	(0.6485)	(0.8089)
UR $\cdot\ \mathbb{I}(LTU)$	-0.162	-1.465	0.223	-0.904	0.377	-1.064	0.298	-0.820	-0.448	-0.860
	(1.3940)	(1.5197)	(1.3146)	(1.4355)	(1.2084)	(1.3312)	(1.0684)	(1.2764)	(0.9661)	(1.1045)
UR $\cdot\ \mathbb{I}(EE$ & switcher)	—	1.690^{**}	—	1.690^{**}	—	1.690^{**}	—	1.690^{**}	—	1.690^{**}
		(0.7457)		(0.7457)		(0.7457)		(0.7457)		(0.7457)
UR $\cdot\ \mathbb{I}(ENE$ & switcher)	—	-3.030^{***}	—	-3.105^{***}	—	-3.367^{***}	—	-3.442^{***}	—	-3.381^{***}
		(0.9856)		(0.9996)		(1.0139)		(1.0431)		(1.0858)
UR $\cdot\ \mathbb{I}(LTU$ & switcher)	—	1.776	—	1.487	—	1.933	—	1.527	—	0.505
		(2.0951)		(1.9504)		(1.7847)		(1.6874)		(1.5259)
Ind. controls	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
$P(\pi_{nu}^{EE} = \pi_{nu}^{ENE})$	0.022	0.000	0.031	0.000	0.040	0.000	0.045	0.000	0.026	0.000
No. observations	375,642	375,641	375,642	375,641	375,642	375,641	375,642	375,641	375,642	375,641

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.

Table B.4: Two separate employment spells, one job, correct IDs. Job ID preserved across contiguous employment spells because individual reports employment for each wave.

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID
1	01/96-04/96	A	09/95	M1-M4	A
2	05/96-08/96	A	09/95	M1	A
3	09/96-12/96	A	09/95	M2-M4	A

Table B.5: Two separate employment spells, one job, incorrect IDs. Job ID information is lost when individual spends an entire wave without employment. At wave 3, the job is incorrectly coded as being a new job and the start date is asked again.

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID
1	01/96-04/96	A	09/95	M1-M4	A
2	05/96-08/96	–	–	none	–
3	09/96-12/96	B	09/95	M1-M4	A

Table B.6: Two separate employment spells, "gap date" falls after reported job start date for job "B". Rule out wave 3 job as "new hire".

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID	Gap date
1	01/96-04/96	A	09/95	M1-M4	A	–
2	05/96-08/96	–	–	none	–	05/96
3	09/96-12/96	B	09/95	M1-M4	A	05/96

Table B.7: Two separate employment spells, "gap date" is prior to reported job start date for job "B". Count wave 3 job as "new hire".

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID	Gap date
1	01/96-04/96	A	09/95	M1-M4	A	–
2	05/96-08/96	–	–	none	–	05/96
3	09/96-12/96	B	08/96	M1-M4	A	05/96

C Model appendix

In this appendix we first close the model by describing firms' capital renting decision and the households' consumption/saving decision (which in turn determine labor productivity and the discount factor), and we define the recursive equilibrium. We then proceed to derive the log-linear equations that describe the first-order dynamics of the model. Most of the derivations are standard or are similar to those in Gertler and Trigari (GT, 2009). We thus focus on non-standard or new derivations. In the second section, we discuss first order-approximations related to the three key decisions in the model: hiring, search intensity and bargaining. In the third section, we derive log-linear expressions for the wage growth of job changers and the shares of job-to-job flows. In the fourth section, we describe the steady state of the labor market and in the fifth section we show how our calibration strategy permits to pin down the key labor market parameters associated with the composition effect. In the sixth section, we derive the loglinear expression for the measured user cost of labor that we discuss in Section 6 of the main text. In the seventh section, we consider lateral job match movements: We show that up to a first order the gains from lateral movements are negligible, which justifies our ruling out this possibility in analyzing the worker's decision problem. In the last section, we tie up a final loose end and define the operator mapping the distribution function from period t to period $t+1$.

C.1 Closing the model

C.1.1 Firms: capital renting and labor productivity

Firms produce output y_t using capital and labor according to a Cobb-Douglas production technology:

$$y_t = z_t k_t^{\zeta} l_t^{1-\zeta}, \tag{26}$$

where k_t is capital, l_t labor in efficiency units and z_t is total factor productivity. Capital is perfectly mobile. Firms rent capital on a period by period basis. As described in the main text, firms add labor through a search and matching process.

The firm's decision problem is to choose capital k_t and the hiring rate $\varkappa_t$ to maximize the discounted stream of profits net recruiting costs, subject to the equations that govern the laws of motion for labor in efficiency units l_t and the quality mix of labor γ_t, and given the expected paths of rental rates r_t and wages w_t. We express the value of each firm $F_t(l_t, \gamma_t, w_t) \equiv F_t$ as

$$F_t = \max_{k_t, \varkappa_t} \{ z_t k_t^{\zeta} l_t^{1-\zeta} - \frac{\kappa}{2} \varkappa_t^2 l_t - w_t l_t - r_t k_t + E_t\{\Lambda_{t,t+1} F_{t+1}\}\},$$

subject to dynamic equations for l_t and γ_t, and given the values of the firm level states (l_t, γ_t, w_t) and the aggregate state vector.

Given constant returns and perfectly mobile capital, the firm's value F_t is homogeneous in l_t. The net effect is that each firm's choice of the capital/labor ratio and the hiring rate is independent of its size. Let J_t be the firm value per efficiency unit of labor and let $\check{k}_t \equiv k_t/l_t$ be its capital labor ratio. Then

$$F_t = J_t \cdot l_t, \tag{27}$$

with $J_t \equiv J_t(\gamma_t, w_t)$ given by

$$J_t = \max_{\check{k}_t, \varkappa_t} \{z_t \check{k}_t^{\zeta} - \frac{\kappa}{2}\varkappa_t^2 - w_t - r_t \check{k}_t + (\rho_t + \varkappa_t) E_t\{\Lambda_{t,t+1} J_{t+1}\}\}, \tag{28}$$

subject to the law of motion for γ_t.

The first order condition for capital renting is

$$r_t = \zeta z_t \check{k}_t^{\zeta - 1}. \tag{29}$$

Given Cobb-Douglas production technology and perfect mobility of capital, $\check{k}_t$ does not vary across firms.

Substituting yields the expression for J_t that appears in the main text, given by

$$J_t = \max_{\varkappa_t} \{a_t - \frac{\kappa}{2}\varkappa_t^2 - w_t + (\rho_t + \varkappa_t) E_t\{\Lambda_{t,t+1} J_{t+1}\}\}, \tag{30}$$

where a_t denotes the current marginal product of labor (i.e., $a_t = (1 - \zeta)/l_t$), which is independent of the firm.

C.1.2 Households: consumption and saving

We adopt the representative family construct, following Merz (1995) and Andolfatto (1996), allowing for perfect consumption insurance. There is a measure of families on the unit interval, each with a measure one of workers. Before making allocating resources to per-capita consumption and savings, the family pools all wage and unemployment income. Additionally, the family owns diversified stakes in firms that pay out profits. The household can then assign consumption $\bar{c}_t$ to members and save in the form of capital $\overline{k}_t$, which is rented to firms at rate r_t and depreciates at the rate δ.

Let Ω_t be the value of the representative household. Then,

$$\Omega_t = \max_{\bar{c}_t, \bar{k}_{t+1}} \{\log(\bar{c}_t) + \beta E_t \Omega_{t+1}\} \tag{31}$$

subject to

$$
\begin{aligned}
&\bar{c}_t + \bar{k}_{t+1} + \frac{\varsigma_0}{1+\eta_\varsigma}\left\{\nu\varsigma_n^{1+\eta_\varsigma}\bar{n}_t + \nu\bar{\varsigma}_{bt}^{1+\eta_\varsigma}\bar{b}_t\right\} \\
&= \bar{w}_t\bar{n}_t + \phi\bar{w}_t\bar{b}_t + (1-\bar{n}_t-\bar{b}_t)u_B + (1-\delta+r_t)\bar{k}_t + T_t + \Pi_t, \qquad (32)
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{n}_{t+1} &= \bar{\rho}_t^n\bar{n}_t + \xi p_t\bar{s}_t \qquad (33) \\
\bar{b}_{t+1} &= \bar{\rho}_t^b\bar{b}_t + \xi\bar{\gamma}_t^m p_t\bar{s}_t \qquad (34)
\end{aligned}
$$

where Π_t are the profits from the household's ownership holdings in firms and T_t are lump sum transfers from the government.[4]

The first-order condition from the household's savings problem gives

$$
1 = (1-\delta+r_t)E_t\{\Lambda_{t,t+1}\} \qquad (35)
$$

where $\Lambda_{t,t+1} \equiv \beta\bar{c}_t/\bar{c}_{t+1}$.

C.1.3 Resource constraint, government policy, and equilibrium

The resource constraint states that the total resource allocation towards consumption, investment, vacancy posting costs, and search costs is equal to aggregate output:

$$
\begin{aligned}
\bar{y}_t &= \bar{c}_t + \bar{k}_{t+1} - (1-\delta)\bar{k}_t \qquad (36) \\
&+\frac{\kappa}{2}\int_i \varkappa_t^2 l_t di + \frac{\varsigma_0}{1+\eta_\varsigma}\left(\nu\varsigma_n^{1+\eta_\varsigma}\bar{n}_t + \nu\bar{\varsigma}_{bt}^{1+\eta_\varsigma}\bar{b}_t\right) \qquad (37)
\end{aligned}
$$

The government funds unemployment benefits through lump-sum transfers:

$$
T_t + \left(1-\bar{n}_t-\bar{b}_t\right)u_B = 0. \qquad (38)
$$

A recursive equilibrium is a solution for (i) a set of functions $\{J_t,\, V_t^n,\, V_t^b, U_t\}$; (ii) the contract wage w_t^*; (iii) the hiring rate $\varkappa_t$; (iv) the subsequent period's wage rate w_{t+1}; (v)

[4] Chodorow-Reich and Karabarbounis (2016) show the introduction of utility from leisure can greatly increase the difficulty of generating sufficient unemployment volatility when the model is calibrated to match the estimated cyclicality of the opportunity cost of employment. For simplicity we do not include utility from leisure, but in ongoing work we show that our model with staggered wage contracting is robust to this critique. Further, we model the cost of search as pecuniary rather than utility. Having a utility cost of search may dampen the procyclicality of search intensity because recessions are times when workers value additional consumption relative to leisure. We leave the quantitative exploration of this mechanism for future research.

the search intensity of a worker in a bad match ς_{bt};(vi) the rental rate on capital r_t; (vii) the average wage, the average contract wage, the average search intensity of workers in bad matches and the average hiring rate, $\bar{w}_t$, $\bar{w}_t^*$, $\bar{\varsigma}_{bt}$ and $\bar{\varkappa}_t$; (viii) the capital labor ratio $\check{k}_t$; (ix) the average consumption and capital, $\bar{c}_t$ and $\bar{k}_{t+1}$; (x) the average employment in good and bad matches, $\bar{n}_t$ and $\bar{b}_t$; (xi) the density function of composition and wages across workers $dG_t(\gamma, w)$; and (xii) a transition function $Q_{t,t+1}$. The solution is such that (i) w_t^* satisfies the Nash bargaining condition; (ii) $\varkappa_t$ satisfies the hiring condition; (iii) w_{t+1} is given by the Calvo process for wages; (iv) ς_{bt} satisfies the first-order condition for search intensity of workers in bad matches; (v) r_t satisfies the first-order condition for capital renting; (vi) $\bar{w}_t = \int_{w,\gamma} w dG_t(\gamma, w)$, $\bar{w}_t^* = \int_{w,\gamma} w_t^*(\gamma) dG_t(\gamma, w)$, $\bar{\varsigma}_{bt} = \int_{w,\gamma} \varsigma_{bt}(\gamma, w) dG_t(\gamma, w)$ and $\bar{\varkappa}_t = \int_{w,\gamma} \varkappa_t(\gamma, w) dG_t(\gamma, w)$; (vii) the rental market for capital clears, $\check{k}_t = \bar{k}_t / \left(\bar{n}_t + \phi \bar{b}_t\right)$; (viii) $\bar{c}_t$ and $\bar{k}_{t+1}$ solve the household problem; (ix) $\bar{n}_t$ and $\bar{b}_t$ evolve according to (33) and (34); (x) the evolution of G_t is consistent with $Q_{t,t+1}$; (xi) $Q_{t,t+1}$ is defined in section C.7 of the appendix.

C.2 Hiring, search intensity and staggered Nash bargaining

Relative to GT, where the only firm-specific state variable was wages, here we must also keep track of composition. As might be expected, there is a non-trivial interplay between composition and wages at the firm level. Composition is inherited from the previous period and influences the wage through the Nash wage bargain; the wage influences next-period composition through hiring and search intensity. We introduce the restriction that good and bad matches have the same steady state job survival rates, which lends considerable analytic and computational tractability to the analysis.

We first state a set of results that will simplify the derivation of the log-linear equations. These include a set of steady state results and approximations of period-ahead firm and worker surpluses at renegotiating firms. We establish how these properties are used to show that the "composition effect" in hiring – wherein firms vary the hiring rate to vary next-period composition – is zero up to a first order. We also briefly discuss the absence of a composition effect in search intensity. We then go over the relevant equations for determining the Nash contract wage: the worker and firm surpluses and the Nash first order condition. We then derive recursive log-linear expressions for the average firm and worker surpluses making use of the surplus approximations previously stated. In doing that, we also derive expressions for the average hiring rate, search intensity, and retention rate at a renegotiating firm. Finally, we prove the steady-state results and the surplus approximations that we invoke for deriving recursive log-linear expressions for the average worker and firm surplus and for linearizing the composition term in hiring.

C.2.1 Some useful results

Let $\tilde{z}$ denote the steady state of variable z_t. Assume that $\varsigma_n = \xi\tilde{\varsigma}_b$, so that $\tilde{\rho}_n = \tilde{\rho}_b = \tilde{\rho}$ (i.e., retention rates of good and bad workers are the same in steady state). Then we obtain the following steady state results:

$$\frac{\partial \gamma_{t+1}}{\partial \varkappa_t} \mid_{ss}= 0 \tag{39}$$

$$\frac{\partial w_t^*(\gamma_t)}{\partial \gamma_t} \mid_{ss}= 0 \tag{40}$$

$$\frac{\partial J_t(\gamma_t, w_t)}{\partial \gamma_t} \mid_{ss}= \frac{\partial H_t(\gamma_t, w_t)}{\partial \gamma_t} \mid_{ss}= \frac{\partial H_t^b(\gamma_t, w_t)}{\partial \gamma_t} \mid_{ss}= 0 \tag{41}$$

Let $\bar{z}_t \equiv \int z_t(\gamma, w)\, dG_t(\gamma, w)$ denote the time t average of a firm-specific variable $z_t(\gamma_t, w_t)$ and note that, up to a first order, $\bar{z}_t = z_t(\bar{\gamma}_t, \bar{w}_t)$. Let $\widehat{z}_t$ denote the log deviation of a variable z_t from its steady state value $\tilde{z}$. The following approximations for the period-ahead firms and worker surpluses hold:

$$\begin{aligned} \widehat{H}_{t+1}\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right) \\ \widehat{H}_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right) + \eta_{Hw}\left(\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right) \end{aligned} \tag{42}$$

$$\begin{aligned} \widehat{J}_{t+1}\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right) &= \widehat{J}_{t+1}\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right) \\ \widehat{J}_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right) &= \widehat{J}_{t+1}\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right) + \eta_{Jw}\left(\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right) \end{aligned} \tag{43}$$

$$\begin{aligned} \widehat{H}_{t+1}^b\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right) &= \widehat{H}_{t+1}^b\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right) \\ \widehat{H}_{t+1}^b\left(\gamma_{t+1}, \bar{w}_t^*\right) &= \widehat{H}_{t+1}^b\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right) + \eta_{H^bw}\left(\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right) \end{aligned} \tag{44}$$

where η_{Hw}, η_{Jw}, and η_{H^bw} are the steady state elasticities of the worker surplus in good matches, H, the firm surplus, J, and the worker surplus in bad matches, H^b, with respect to the wage.

We use results (39)-(44) to prove that the "composition effect" of hiring is zero up to a first order and to solve for the average contract wage up to a first order, which in turn requires deriving recursive loglinear equations of the firm and worker surpluses. We will invoke these results in the following subsections and then prove them at the end of the section.

C.2.2 Hiring

In the main text, we derive the first order condition for hiring, $\varkappa_t$. Given that next period wage equals this period wage w_t with probability λ and next period contract wage $w^*_{t+1}\left(\gamma_{t+1}\right)$ with probability $1-\lambda$, we can write the hiring condition at a firm with composition γ_t and wage w_t as follows:

$$\kappa\varkappa_t\left(\gamma_t, w_t\right) = E_t\{\Lambda_{t,t+1}\left[\lambda J_{t+1}\left(\gamma_{t+1}, w_t\right) + (1-\lambda) J_{t+1}\left(\gamma_{t+1}, w^*_{t+1}\left(\gamma_{t+1}\right)\right)\right]\} + \omega_t\left(\gamma_t, w_t\right)$$

where the second term represents a composition term in hiring:

$$\begin{aligned}\omega_t\left(\gamma_t, w_t\right) = {} & \left[\rho_t\left(\gamma_t\right) + \varkappa_t\left(\gamma_t, w_t\right)\right] \times \\ & E_t\left\{\Lambda_{t,t+1}\left[\lambda\frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}} + (1-\lambda)\frac{\partial J_{t+1}(\gamma_{t+1}, w^*_{t+1}\left(\gamma_{t+1}\right))}{\partial\gamma_{t+1}}\right.\right. \\ & \left.\left. +(1-\lambda)\frac{\partial J_{t+1}(\gamma_{t+1}, w^*_{t+1}\left(\gamma_{t+1}\right))}{\partial w^*_{t+1}\left(\gamma_{t+1}\right)}\frac{\partial w^*_{t+1}\left(\gamma_{t+1}\right)}{\partial\gamma_{t+1}}\right]\frac{\partial\gamma_{t+1}}{\partial\varkappa_t}\right\}\end{aligned}$$

The firm cares about period-ahead composition for the implied period-ahead retention rate of a unit of labor quality (represented by the first two terms in square brackets) and through possible effects of period-ahead firm composition on future renegotiated wages (the third term).

Since we will prove that $\partial J/\partial\gamma$, $\partial w^*\left(\gamma\right)/\partial\gamma$, $\partial\gamma'/\partial\varkappa$ are all equal to 0 in the steady state, it follows that up to a first order the composition term $\omega_t\left(\gamma_t, w_t\right) = 0$.

C.2.3 Search intensity

In the main text, we also derive the first order condition for search intensity, ς_{bt}. Similarly to hiring, we can write the search intensity condition at a firm with composition γ_t and wage w_t as

$$\varsigma_0\varsigma_{bt}^{\eta_\varsigma}(\gamma_t, w_t) = p^n_t E_t\left\{\Lambda_{t,t+1}\left[\bar{H}_{t+1} - \lambda H^b_{t+1}(\gamma_{t+1}, w_t) - (1-\lambda) H^b_{t+1}(\gamma_{t+1}, w^*_{t+1}\left(\gamma_{t+1}\right))\right]\right\}$$

One difference with respect to the hiring condition is that no composition term in search intensity is present. A worker deciding on her own search intensity does not internalize the effect her choice has on the average search intensity of workers employed in bad matches and thus on period-ahead firm composition. This happens because the firm employs a continuum of workers and each worker behaves atomistically.

C.2.4 Staggered Nash bargaining

Consider the problem of a firm and its workers employed in good matches renegotiating a new contract wage, $w_t^*(\gamma_t)$. For any composition γ_t, we can write the surplus of workers in good matches $H_t(\gamma_t, w_t^*(\gamma_t))$ as

$$\begin{aligned} H_t(\gamma_t, w_t^*(\gamma_t)) &= w_t^*(\gamma_t) - u_B - \nu c(\varsigma_n) + E_t\left\{\Lambda_{t,t+1}\left[\nu\varsigma_n p_t \bar{H}_{t+1}^a - p_t \bar{H}_{t+1}^a\right]\right\} \\ &\quad +\nu(1-\varsigma_n p_t) E_t\left\{\Lambda_{t,t+1}\left[\lambda H_{t+1}\left(\gamma_{t+1}, w_t^*(\gamma_t)\right)\right.\right. \\ &\quad \left.\left. +\ (1-\lambda) H_{t+1}\left(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1})\right)\right]\right\} \end{aligned}$$

with

$$\begin{aligned} \bar{H}_t^a &\equiv \xi \bar{H}_t + (1-\xi)\bar{H}_t^b \\ \bar{H}_t &\equiv \bar{V}_t^n - U_t \\ \bar{H}_t^b &\equiv \bar{V}_t^b - U_t \end{aligned}$$

Similarly, we can write firm surplus $J_t(\gamma_t, w_t^*(\gamma_t))$ as

$$\begin{aligned} J_t(\gamma_t, w_t^*(\gamma_t)) &= a_t - w_t^*(\gamma_t) - \frac{\kappa}{2}\varkappa_t(\gamma_t, w_t^*(\gamma_t))^2 \\ &\quad +[\rho_t(\gamma_t) + \varkappa_t(\gamma_t, w_t^*(\gamma_t))] \times \\ &\quad E_t\left\{\Lambda_{t,t+1}\left[\lambda J_{t+1}\left(\gamma_{t+1}, w_t^*(\gamma_t)\right) + (1-\lambda) J_{t+1}\left(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1})\right)\right]\right\} \end{aligned}$$

with

$$\begin{aligned} \kappa\varkappa_t(\gamma_t, w_t^*(\gamma_t)) &= E_t\left\{\Lambda_{t,t+1}\left[\lambda J_{t+1}\left(\gamma_{t+1}, w_t^*(\gamma_t)\right) + (1-\lambda) J_{t+1}\left(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1})\right)\right]\right\} \\ &\quad +\omega_t(\gamma_t, w_t^*(\gamma_t)) \end{aligned}$$

and

$$a_t \equiv (1-\zeta) z_t \check{k}_t^\alpha$$

In the main text, we write the Nash bargaining condition:

$$\chi_t(\gamma_t, w_t^*(\gamma_t)) J_t(\gamma_t, w_t^*(\gamma_t)) = (1-\chi_t(\gamma_t, w_t^*(\gamma_t))) H_t(\gamma_t, w_t^*(\gamma_t)$$

where

$$\chi_t(\gamma_t, w_t^*(\gamma_t)) = \frac{\eta}{\eta + (1-\eta)\mu_t(\gamma_t, w_t^*(\gamma_t))/\epsilon_t(\gamma_t, w_t^*(\gamma_t))}$$

with

$$\epsilon_t(\gamma_t, w_t^*(\gamma_t)) = \frac{\partial H_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} \quad \text{and} \quad \mu_t(\gamma_t, w_t^*(\gamma_t)) = \frac{\partial J_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)}$$

As we discuss in the text, the Nash condition is a variation of the conventional sharing rule, where the relative weight χ depends not only on the worker's bargaining power η, but also on the differential firm/worker horizon, reflected by the term μ/ϵ. While the horizon effect is interesting from a theoretical perspective, GT shows that it is not quantitatively important. Since this greatly enhances model tractability, in the quantitative analysis we abstract from the horizon effect by having the relative weight χ fixed at η, thus effectively working with the following simplified sharing rule:

$$\eta J_t\left(\gamma_t, w_t^*\left(\gamma_t\right)\right) = (1-\eta) H_t\left(\gamma_t, w_t^*\left(\gamma_t\right)\right)$$

To solve for the average contract wage, $\bar{w}_t^*$, we now derive loglinear recursive expressions for both the average surplus of workers in good matches, $H_t\left(\bar{\gamma}_t, \bar{w}_t^*\right)$, and the average firm surplus, $J_t\left(\bar{\gamma}_t, \bar{w}_t^*\right)$, at a renegotiating firm. As it will become clear, we will also need expressions for renegotiating firms of the average hiring rate, $\varkappa_t\left(\bar{\gamma}_t, \bar{w}_t^*\right)$, the average retention rate, $\rho_t\left(\bar{\gamma}_t, \bar{w}_t^*\right)$, the average search intensity, $\varsigma_{bt}\left(\bar{\gamma}_t, \bar{w}_t^*\right)$, and the average worker surplus in bad matches, $H_t^b\left(\bar{\gamma}_t, \bar{w}_t^*\right)$.

C.2.4.1 Average surplus of workers in good matches at a renegotiating firm

We start deriving a loglinear recursive expression for the average surplus of workers in good matches at a renegotiating firm.

The nonlinear expression for $H_t\left(\bar{\gamma}_t, \bar{w}_t^*\right)$ is given by

$$\begin{aligned} H_t\left(\bar{\gamma}_t, \bar{w}_t^*\right) &= w_t^*\left(\bar{\gamma}_t\right) - u_B - \nu c(\varsigma_n) + E_t\left\{\Lambda_{t,t+1}\left[\nu\varsigma_n p_t \bar{H}_{t+1}^a - p_t \bar{H}_{t+1}^a\right]\right\} \\ &\quad + \nu\left(1-\varsigma_n p_t\right)\lambda E_t\left\{\Lambda_{t,t+1} H_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right)\right\} \\ &\quad + \nu\left(1-\varsigma_n p_t\right)(1-\lambda) E_t\left\{\Lambda_{t,t+1} H_{t+1}\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right)\right\} \end{aligned}$$

where γ_{t+1} denotes next period composition at the firm and $w_{t+1}^*\left(\gamma_{t+1}\right)$ next period contract wage in case of renegotiation.

Loglinearizing and rearranging, we obtain:

$$\begin{aligned} \widehat{H}_t\left(\bar{\gamma}_t, \bar{w}_t^*\right) &= \left(\tilde{w}/\tilde{H}\right)\widehat{\bar{w}}_t^* + (\nu-\tilde{\rho})\beta\left(\tilde{H}^a/\tilde{H}\right) E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}^a\right\} \\ &\quad - \tilde{\rho}\beta\left(\tilde{H}^a/\tilde{H}\right) E_t\left\{\widehat{p}_t + \widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}^a\right\} - (\nu-\tilde{\rho})\beta\left(1-\left(\tilde{H}^a/\tilde{H}\right)\right)\widehat{p}_t \\ &\quad + \tilde{\rho}\beta E_t\left\{\widehat{\Lambda}_{t,t+1} + \lambda\widehat{H}_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right) + (1-\lambda)\widehat{H}_{t+1}\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right)\right\} \end{aligned}$$

Using now the approximation in equation (42) to substitute out terms in the last line and rearranging, we obtain a recursive loglinear expression for $H_t(\bar{\gamma}_t, \bar{w}_t^*)$, as follows:

$$\begin{aligned}
\widehat{H}_t(\bar{\gamma}_t, \bar{w}_t^*) &= \left(\tilde{w}/\tilde{H}\right)\left[\widehat{\bar{w}}_t^* + \tilde{\rho}\lambda\beta\tilde{\varepsilon}E_t\left\{\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right\}\right] \\
&+(\nu - \tilde{\rho})\beta\left(\tilde{H}^a/\tilde{H}\right)E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}^a\right\} \\
&-\tilde{p}\beta\left(\tilde{H}^a/\tilde{H}\right)E_t\left\{\widehat{p}_t + \widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}^a\right\} \\
&-(\nu - \tilde{\rho})\beta\left(1 - \left(\tilde{H}^a/\tilde{H}\right)\right)\widehat{p}_t \\
&+\tilde{\rho}\beta E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*)\right\}
\end{aligned}$$

where $\tilde{\varepsilon} \equiv \partial H/\partial w$.

C.2.4.2 Average surplus of firms at a renegotiating firm

Here we derive a loglinear recursive expression for the average firm surplus at renegotiating firms.

The nonlinear expression for $J_t(\bar{\gamma}_t, \bar{w}_t^*)$ is given by

$$\begin{aligned}
J_t(\bar{\gamma}_t, \bar{w}_t^*) &= a_t - \bar{w}_t^* + \frac{\kappa}{2}\varkappa_t(\bar{\gamma}_t, \bar{w}_t^*)^2 \\
&+\rho_t(\bar{\gamma}_t, \bar{w}_t^*)\lambda E_t\left\{\Lambda_{t,t+1}J_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right)\right\} \\
&+\rho_t(\bar{\gamma}_t, \bar{w}_t^*)(1-\lambda)E_t\left\{\Lambda_{t,t+1}J_{t+1}\left(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1})\right)\right\}
\end{aligned}$$

where, as for $H_t(\bar{\gamma}_t, \bar{w}_t^*)$, γ_{t+1} denotes next period composition and $w_{t+1}^*\left(\gamma_{t+1}\right)$ next period contract wage, and where we have dropped the term $\omega_t(\bar{\gamma}_t, \bar{w}_t^*)$ that is zero up to a first order.

Loglinearizing and rearranging, we obtain:

$$\begin{aligned}
\widehat{J}_t(\bar{\gamma}_t, \bar{w}_t^*) &= \left(\tilde{a}/\tilde{J}\right)\widehat{a}_t - \left(\tilde{w}/\tilde{J}\right)\widehat{\bar{w}}_t^* + (1-\tilde{\rho})\beta\widehat{\varkappa}_t(\bar{\gamma}_t, \bar{w}_t^*) \\
&+\tilde{\rho}\beta\lambda E_t\left\{\widehat{\rho}_t(\bar{\gamma}_t, \bar{w}_t^*) + \widehat{\Lambda}_{t,t+1} + \widehat{J}_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right)\right\} \\
&+\tilde{\rho}\beta(1-\lambda)E_t\left\{\widehat{\rho}_t(\bar{\gamma}_t, \bar{w}_t^*) + \widehat{\Lambda}_{t,t+1} + \widehat{J}_{t+1}\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right)\right\}
\end{aligned}$$

Using now the approximation in equation (43) to substitute out terms in the last two lines and rearranging, we obtain a recursive loglinear expression for $J_t(\bar{\gamma}_t, \bar{w}_t^*)$, as follows:

$$\begin{aligned}
\widehat{J}_t(\bar{\gamma}_t, \bar{w}_t^*) &= \left(\tilde{a}/\tilde{J}\right)\widehat{a}_t - \left(\tilde{w}/\tilde{J}\right)E_t\left\{\widehat{\bar{w}}_t^* + \tilde{\rho}\lambda\beta\tilde{\mu}\left(\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right)\right\} + (1-\tilde{\rho})\beta\widehat{\varkappa}_t(\bar{\gamma}_t, \bar{w}_t^*) \\
&+\tilde{\rho}\beta E_t\left\{\widehat{\rho}_t(\bar{\gamma}_t, \bar{w}_t^*) + \widehat{\Lambda}_{t,t+1} + \widehat{J}_t\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right)\right\}
\end{aligned}$$

where $\tilde{\mu} \equiv \partial J/\partial w$.

We note that $\widehat{J}_t(\bar{\gamma}_t, \bar{w}_t^*)$ depends on $\widehat{\varkappa}_t(\bar{\gamma}_t, \bar{w}_t^*)$ and $\widehat{\rho}_t(\bar{\gamma}_t, \bar{w}_t^*)$. The latter, in turn, depends on $\widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*)$ since

$$\widehat{\rho}_t(\bar{\gamma}_t, \bar{w}_t^*) = -\frac{\nu - \tilde{\rho}}{\tilde{\rho}}\left(\widehat{p}_t + \widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*)\right)$$

We thus proceed to derive loglinear expressions for $\varkappa_t(\bar{\gamma}_t, \bar{w}_t^*)$ and $\varsigma_{bt}(\bar{\gamma}_t, \bar{w}_t^*)$.

C.2.4.3 Average hiring rate at a renegotiating firm The nonlinear expression for $\varkappa_t(\bar{\gamma}_t, \bar{w}_t^*)$ is given by

$$\kappa\varkappa_t(\bar{\gamma}_t, \bar{w}_t^*) = E_t\{\Lambda_{t,t+1}\left[\lambda J_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right) + (1-\lambda) J_{t+1}\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right)\right]\}$$

where, as before, we have dropped the term $\omega_t(\bar{\gamma}_t, \bar{w}_t^*)$ that is zero up to a first order.

Loglinearizing and rearranging, we obtain:

$$\widehat{\varkappa}_t(\bar{\gamma}_t, \bar{w}_t^*) = E_t\left\{\widehat{\Lambda}_{t,t+1} + \lambda\widehat{J}_{t+1}\left(\gamma_{t+1}, \bar{w}_t^*\right) + (1-\lambda)\widehat{J}_{t+1}\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right)\right\}$$

Using again the approximation in equation (43), we obtain

$$\widehat{\varkappa}_t(\bar{\gamma}_t, \bar{w}_t^*) = -\left(\tilde{w}/\tilde{J}\right)\lambda\tilde{\mu}\left(\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right) + E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{J}_t\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right)\right\}$$

C.2.4.4 Average search intensity at a renegotiating firm The nonlinear expression for $\varsigma_{bt}(\bar{\gamma}_t, \bar{w}_t^*)$ is given by

$$\begin{aligned}
\varsigma_0\varsigma_{bt}^{\eta_\varsigma}(\bar{\gamma}_t, \bar{w}_t^*) &= p_t^n E_t\left\{\Lambda_{t,t+1}\bar{H}_{t+1}\right\} \\
&\quad - p_t^n\lambda E_t\left\{\Lambda_{t,t+1}H_{t+1}^b(\gamma_{t+1}, \bar{w}_t^*)\right\} \\
&\quad - p_t^n(1-\lambda)E_t\left\{\Lambda_{t,t+1}H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right))\right\}
\end{aligned}$$

and the loglinear version is given by

$$\begin{aligned}
\eta_\varsigma\widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*) &= E_t\left\{\widehat{p}_t + \widehat{\Lambda}_{t,t+1} + \left(\tilde{H}/\left(\tilde{H} - \tilde{H}^b\right)\right)\widehat{\bar{H}}_{t+1}\right\} \\
&\quad - \left(\tilde{H}^b/\left(\tilde{H} - \tilde{H}^b\right)\right)E_t\left\{\lambda\widehat{H}_{t+1}^b(\gamma_{t+1}, \bar{w}_t^*) + (1-\lambda)\widehat{H}_{t+1}^b(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right))\right\}
\end{aligned}$$

Using now the approximation in equation (44), we obtain

$$\begin{aligned}
\eta_\varsigma\widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*) &= -\lambda\left(\tilde{w}/\left(\tilde{H} - \tilde{H}^b\right)\right)\tilde{\epsilon}^b E_t\left\{\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right\} \\
&\quad + E_t\left\{\widehat{p}_t + \widehat{\Lambda}_{t,t+1} + \left(1/\left(\tilde{H} - \tilde{H}^b\right)\right)\left(\tilde{H}\widehat{\bar{H}}_{t+1} - \tilde{H}^b\widehat{H}_t^b\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right)\right)\right\}
\end{aligned}$$

where $\tilde{\varepsilon}^b \equiv \partial H^b/\partial w$.

Since $\widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*)$ depends on $\widehat{H}_t^b(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*)$, we finally derive a recursive loglinear expression for $H_t^b(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*)$.

C.2.4.5 Average surplus of workers in bad matches at a renegotiating firm

The nonlinear expression for $H_t^b(\bar{\gamma}_t, \bar{w}_t^*)$ is given by

$$\begin{aligned}
H_t^b(\bar{\gamma}_t, \bar{w}_t^*) \;&=\; \phi w_t^*(\bar{\gamma}_t) - u_B - \nu c(\varsigma_{bt}(\bar{\gamma}_t, \bar{w}_t^*)) \\
&+E_t\left\{\Lambda_{t,t+1}\left[\nu\varsigma_{bt}(\bar{\gamma}_t, \bar{w}_t^*)\, p_t^n \bar{H}_{t+1} - p_t \bar{H}_{t+1}^a\right]\right\} \\
&+\nu\left(1-\varsigma_{bt}(\bar{\gamma}_t, \bar{w}_t^*)\, p_t^n\right)\lambda E_t\left\{\Lambda_{t,t+1} H_{t+1}^b\left(\gamma_{t+1}, \bar{w}_t^*\right)\right\} \\
&+\nu\left(1-\varsigma_{bt}(\bar{\gamma}_t, \bar{w}_t^*)\, p_t^n\right)(1-\lambda)\, E_t\left\{\Lambda_{t,t+1} H_{t+1}^b\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right)\right\}
\end{aligned}$$

Loglinearizing and rearranging, we obtain:

$$\begin{aligned}
\widehat{H}_t^b(\bar{\gamma}_t, \bar{w}_t^*) \;&=\; \left(\tilde{w}/\tilde{H}\right)\phi\widehat{\bar{w}}_t^* - \left(\nu\varsigma_0\tilde{\varsigma}_b^{1+\eta_\varsigma}/\tilde{H}^b\right)\widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*) \\
&+(\nu-\tilde{\rho})\beta\left(\tilde{H}/\tilde{H}^b\right)E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}\right\} \\
&-\tilde{\rho}\beta\left(\tilde{H}^a/\tilde{H}^b\right)E_t\left\{\widehat{p}_t + \widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}^a\right\} \\
&-(\nu-\tilde{\rho})\beta\left(1-\left(\tilde{H}^a/\tilde{H}^b\right)\right)\left(\widehat{p}_t + \widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*)\right) \\
&+\tilde{\rho}\beta\lambda E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}^b\left(\gamma_{t+1}, \bar{w}_t^*\right)\right\} \\
&+\tilde{\rho}\beta(1-\lambda)\, E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}^b\left(\gamma_{t+1}, w_{t+1}^*\left(\gamma_{t+1}\right)\right)\right\}
\end{aligned}$$

Using again equation (44) and rearranging, we obtain a recursive loglinear expression for $H_t^b(\bar{\gamma}_t, \bar{w}_t^*)$, as follows:

$$\begin{aligned}
\widehat{H}_t^b(\bar{\gamma}_t, \bar{w}_t^*) \;&=\; \left(\tilde{w}/\tilde{H}\right)E_t\left\{\phi\widehat{\bar{w}}_t^* + \tilde{\rho}\lambda\beta\tilde{\varepsilon}^b\left(\widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t+1}^*\right)\right\} \\
&-\left(\nu\varsigma_0\tilde{\varsigma}^{1+\eta_\varsigma}/\tilde{H}^b\right)\widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*) \\
&+(\nu-\tilde{\rho})\beta\left(\tilde{H}/\tilde{H}^b\right)E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}\right\} \\
&-\tilde{\rho}\beta\left(\tilde{H}^a/\tilde{H}^b\right)E_t\left\{\widehat{p}_t + \widehat{\Lambda}_{t,t+1} + \widehat{\bar{H}}_{t+1}^a\right\} \\
&-(\nu-\tilde{\rho})\beta\left(1-\left(\tilde{H}^a/\tilde{H}^b\right)\right)\left(\widehat{p}_t + \widehat{\varsigma}_{bt}(\bar{\gamma}_t, \bar{w}_t^*)\right) \\
&+\tilde{\rho}\beta E_t\left\{\widehat{\Lambda}_{t,t+1} + \widehat{H}_t^b\left(\bar{\gamma}_{t+1}, \bar{w}_{t+1}^*\right)\right\}
\end{aligned}$$

where $\tilde{\varepsilon} \equiv \partial H/\partial w$.

C.2.5 Derivation of steady-state results

We now derive the steady-state results invoked at the beginning of the appendix. In particular, we show that $\partial w_t^* (\gamma_t) / \partial \gamma_t$ equals zero in the steady state. In doing that, we also show that $\partial \gamma_{t+1} / \partial \varkappa_t$, $\partial J_t (\gamma_t, w_t) / \partial \gamma_t$, $\partial H_t (\gamma_t, w_t) / \partial \gamma_t$, $\partial H_t^b (\gamma_t, w_t) / \partial \gamma_t$, $\partial \varsigma_{bt} (\gamma_t, w_t) / \partial \gamma_t$, and $\partial \rho_t (\gamma_t, w_t) / \partial \gamma_t$ are all equal to zero in the steady state.

More precisely, we solve a system of steady-state equations in the partial derivatives of several variables with respect to the two firm-level state variables: current composition and current wage. We show that in the steady state the outcome of the current wage bargain is independent of the firm's current composition, as long as the relevant parties believe that the same is true of the outcome of future wage bargains.[5]

In what follows, we first derive the relevant equations belonging to the system; evaluate them at steady state; and finally show that the solution is such that $\partial w_t^* (\gamma_t) / \partial \gamma_t \mid_{ss} = 0$.

C.2.5.1 Effect of composition on contract wage Consider a renegotiating firm with composition γ_t and contract wage $w_t^* (\gamma_t)$. Define

$$\digamma_t (\gamma_t, w_t^* (\gamma_t)) \equiv \eta J_t(\gamma_t, w_t^* (\gamma_t)) - (1 - \eta) H_t(\gamma_t, w_t^* (\gamma_t))$$

Since $\digamma_t (\gamma_t, w_t^* (\gamma_t)) = 0$ by the surplus sharing condition, we have

$$\frac{\partial w_t^* (\gamma_t)}{\partial \gamma_t} = - \frac{\partial \digamma_t (\gamma_t, w_t^* (\gamma_t)) / \partial \gamma_t}{\partial \digamma_t (\gamma_t, w_t^* (\gamma_t)) / \partial w_t^* (\gamma_t)}$$

where

$$\frac{\partial \digamma_t (\gamma_t, w_t^* (\gamma_t))}{\partial \gamma_t} = \eta \frac{\partial J_t (\gamma_t, w_t^* (\gamma_t))}{\partial \gamma_t} - (1 - \eta) \frac{\partial H_t (\gamma_t, w_t^* (\gamma_t))}{\partial \gamma_t}$$
$$\frac{\partial \digamma_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)} = \eta \frac{\partial J_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)} - (1 - \eta) \frac{\partial H_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)}$$

Evaluating at steady state:

$$\frac{\partial w^*(\gamma)}{\partial \gamma} = - \frac{\eta \partial J / \partial \gamma - (1 - \eta) \partial H / \partial \gamma}{\eta \partial J / \partial w - (1 - \eta) \partial H / \partial w} \tag{S1}$$

which gives us the first equation of the system.

C.2.5.2 Effect of composition and wages on worker surplus in good matches We then obtain expressions for $\partial H / \partial \gamma$ and $\partial H / \partial w$. For any given composition γ_t and wage

[5] While in principle there may be other self-fulfilling solutions in which firm's composition matters to current wages in the steady state simply because the parties believe it will matter in the future, we believe that our fundamentals-based solution is most natural, given the environment.

w_t, the worker surplus in a good match is

$$\begin{aligned}H_t(\gamma_t, w_t) &= w_t - u_B - \nu c(\varsigma_n) \\ &+ E_t\left\{\Lambda_{t,t+1}\left[\nu\varsigma_n p_t \bar{H}^a_{t+1} - p_t \bar{H}^a_{t+1}\right]\right\} \\ &+ \nu(1-\varsigma_n p_t) E_t\left\{\Lambda_{t,t+1}\left[\lambda H_{t+1}(\gamma_{t+1}, w_t)\right.\right. \\ &+ \left.\left.(1-\lambda) H_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)\right]\right\}\end{aligned}$$

We can write $\partial H_t(\gamma_t, w_t)/\partial\gamma_t$ as follows

$$\begin{aligned}\frac{\partial H_t(\gamma_t, w_t)}{\partial\gamma_t} &= \nu(1-\varsigma_n p_t) E_t\left\{\Lambda_{t,t+1}\left[\lambda\frac{\partial H_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}}\right.\right. \\ &+ (1-\lambda)\frac{\partial H_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)}{\partial\gamma_{t+1}} \\ &+ \left.\left.(1-\lambda)\frac{\partial H_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)}{\partial w^*_{t+1}(\gamma_{t+1})}\frac{\partial w^*_{t+1}(\gamma_{t+1}))}{\partial\gamma_{t+1}}\right] \times \frac{d\gamma_{t+1}}{d\gamma_t}\right\}\end{aligned}$$

Evaluating at steady state gives

$$\frac{\partial H}{\partial\gamma}\left(1-\rho\beta\frac{d\gamma'}{d\gamma}\right) = \rho\beta(1-\lambda)\frac{\partial H}{\partial w}\frac{\partial w^*(\gamma)}{\partial\gamma}\frac{d\gamma'}{d\gamma} \quad \text{(S2)}$$

We now turn to $\partial H/\partial w$. We can write $\partial H_t(\gamma_t, w_t)/\partial w_t$ as follows

$$\begin{aligned}\frac{\partial H_t(\gamma_t, w_t)}{\partial w_t} &= 1 + \nu(1-\varsigma_n p_t) E_t\left\{\Lambda_{t,t+1}\lambda\frac{\partial H_{t+1}(\gamma_{t+1}, w_t)}{\partial w_t}\right\} \\ &+ \nu(1-\varsigma_n p_t) E_t\left\{\Lambda_{t,t+1}\left[\lambda\frac{\partial H_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}}\right.\right. \\ &+ (1-\lambda)\frac{\partial H_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)}{\partial\gamma_{t+1}} \\ &+ \left.\left.(1-\lambda)\frac{\partial H_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)}{\partial w^*_{t+1}(\gamma_{t+1})}\frac{\partial w^*_{t+1}(\gamma_{t+1})}{\partial\gamma_{t+1}}\right] \times \frac{d\gamma_{t+1}}{dw_t}\right\}\end{aligned}$$

Evaluating at steady state gives the third equation of the system:

$$\frac{\partial H}{\partial w}\left(1-\rho\beta\lambda-\rho\beta(1-\lambda)\frac{\partial w^*(\gamma)}{\partial\gamma}\frac{d\gamma'}{dw}\right) = 1 + \rho\beta\frac{\partial H}{\partial\gamma}\frac{d\gamma'}{dw} \quad \text{(S3)}$$

C.2.5.3 Effect of composition and wages on firm surplus We now turn to $\partial J/\partial\gamma$ and $\partial J/\partial w$. For any given composition γ_t and wage w_t, the firm surplus is given by

$$\begin{aligned} J_t(\gamma_t, w_t) &= a_t - w_t - \frac{\kappa}{2}\varkappa_t(\gamma_t, w_t)^2 \\ &+ [\rho_t(\gamma_t, w_t) + \varkappa_t(\gamma_t, w_t)] \times \\ &E_t\left\{\Lambda_{t,t+1}\left[\lambda J_{t+1}(\gamma_{t+1}, w_t)\right.\right. \\ &\left.\left. + \ (1-\lambda) J_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)\right]\right\} \end{aligned}$$

which gives $\partial J_t(\gamma_t, w_t)/\partial\gamma_t$ as follows

$$\begin{aligned} \frac{\partial J_t(\gamma_t, w_t)}{\partial\gamma_t} &= \frac{d\rho_t}{d\gamma_t} \times E_t\left\{\Lambda_{t,t+1}\left[\lambda J_{t+1}(\gamma_{t+1}, w_t) + (1-\lambda) J_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)\right]\right\} \\ &+ [\rho_t(\gamma_t, w_t) + \varkappa_t(\gamma_t, w_t)] \times \\ &E_t\left\{\Lambda_{t,t+1}\left[\lambda\frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}}\right.\right. \\ &+ (1-\lambda)\frac{\partial J_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)}{\partial\gamma_{t+1}} \\ &\left.\left. + \ (1-\lambda)\frac{\partial J_{t+1}\left(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})\right)}{\partial w^*_{t+1}(\gamma_{t+1})}\frac{\partial w^*_{t+1}(\gamma_{t+1})}{\partial\gamma_{t+1}}\right] \times \frac{d\gamma_{t+1}}{d\gamma_t}\right\} \end{aligned}$$

where $J_t(\gamma_t, w_t)$ is maximized with respect to $\varkappa_t(\gamma_t, w_t)$, so that in taking the derivative with respect to γ_t, we can hold $\varkappa_t(\gamma_t, w_t)$ fixed at its optimal value.

Evaluating at steady state gives

$$\frac{\partial J}{\partial\gamma}\left(1 - \beta\frac{d\gamma'}{d\gamma}\right) = \frac{d\rho}{d\gamma}\beta J + \beta(1-\lambda)\frac{\partial J}{\partial w}\frac{\partial w^*(\gamma)}{\partial\gamma}\frac{d\gamma'}{d\gamma} \tag{S4}$$

We can then write $\partial J_t(\gamma_t, w_t)/\partial w_t$ as

$$\begin{aligned} \frac{\partial J_t(\gamma_t, w_t)}{\partial w_t} &= -1 + [\rho_t(\gamma_t, w_t) + \varkappa_t(\gamma_t, w_t)]\lambda E_t\left\{\Lambda_{t,t+1}\frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial w_t}\right\} \\ &+\varphi_t(\gamma_t, w_t) \end{aligned}$$

$$
\begin{aligned}
\varphi_t(\gamma_t, w_t) \quad = \quad & [\rho_t(\gamma_t, w_t) + \varkappa_t(\gamma_t, w_t)] E_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right.\right. \\
& + (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w^*_{t+1})}{\partial \gamma_{t+1}} \\
& \left.\left. + (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w^*_{t+1})}{\partial w^*_{t+1}} \frac{\partial w^*_{t+1}}{\partial \gamma_{t+1}} \right] \frac{d\gamma_{t+1}}{dw_t} \right. \\
& + \frac{d\rho_t}{dw_t} E_t \left\{ \Lambda_{t,t+1} \left[\lambda J_{t+1}(\gamma_{t+1}, w_t) \right.\right. \\
& \left.\left. + (1-\lambda) J_{t+1}(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1})) \right] \right\}
\end{aligned}
$$

where we have used the envelope condition.

Evaluating at steady state gives

$$
\frac{\partial J}{\partial w}\left(1 - \lambda\beta - \beta(1-\lambda)\frac{\partial w^*(\gamma)}{\partial \gamma}\frac{d\gamma'}{dw}\right) = -1 + \beta\frac{\partial J}{\partial \gamma}\frac{d\gamma'}{dw} + \frac{d\rho}{dw}\beta J \tag{S5}
$$

C.2.5.4 Effect of composition and wages on the retention rate Here we derive expressions for $d\rho/d\gamma$ and $d\rho/dw$. For any given composition γ_t and wage w_t, the average retention rate is

$$
\rho_t(\gamma_t, w_t) = \frac{\rho^n_t + \phi\gamma_t\rho^b_t(\gamma_t, w_t)}{1 + \phi\gamma_t}
$$

where

$$
\rho^b_t(\gamma_t, w_t) = \nu\left(1 - \varsigma_{bt}(\gamma_t, w_t)\, p^n_t\right)
$$

The derivative of ρ_t with respect to γ_t is

$$
\frac{d\rho_t}{d\gamma_t} = \frac{\partial\rho_t}{\partial\gamma_t} + \frac{\partial\rho_t}{\partial\rho^b_t}\frac{\partial\rho^b_t}{\partial\varsigma_{bt}}\frac{\partial\varsigma_{bt}}{\partial\gamma_t}
$$

and the derivative with respect to w_t is

$$
\frac{d\rho_t}{dw_t} = \frac{\partial\rho_t}{\partial w_t} + \frac{\partial\rho_t}{\partial\rho^b_t}\frac{\partial\rho^b_t}{\partial\varsigma_{bt}}\frac{\partial\varsigma_{bt}}{\partial w_t}
$$

Let us consider each relevant term. We have

$$
\frac{\partial\rho_t}{\partial\gamma_t} = \frac{\phi\rho^b_t(1+\phi\gamma_t) - \phi\left(\rho^n_t + \phi\gamma_t\rho^b_t\right)}{(1+\phi\gamma_t)^2} = \frac{\phi\left(\rho^b_t - \rho^n_t\right)}{(1+\phi\gamma_t)^2}
$$

$$
\frac{\partial\rho_t}{\partial\rho^b_t} = \frac{\phi\gamma_t}{1+\phi\gamma_t}
$$

$$\frac{\partial \rho_t^b}{\partial \varsigma_{bt}} = -\nu p_t^n$$

$$\frac{\partial \rho_t}{\partial w_t} = 0$$

Evaluating terms at steady state, using $\tilde{\rho}^b = \tilde{\rho}^n$, and substituting, we obtain

$$\frac{d\rho}{d\gamma} = -\frac{\phi\gamma}{1+\phi\gamma}\nu p^n \frac{\partial \varsigma_b}{\partial \gamma} \tag{S6}$$

$$\frac{d\rho}{dw} = -\frac{\phi\gamma}{1+\phi\gamma}\nu p^n \frac{\partial \varsigma_b}{\partial w} \tag{S7}$$

C.2.5.5 Effect of composition and wage on future composition We turn to expressions for $d\gamma'/d\gamma$ and $d\gamma'/dw$. Composition evolves as

$$\gamma_{t+1}(\gamma_t, w_t) = \frac{\rho_t^b(\gamma_t, w_t)\,\gamma_t / (1+\phi\gamma_t) + \varkappa_t(\gamma_t, w_t)\,\bar{\gamma}_t^h / \left(1+\phi\bar{\gamma}_t^h\right)}{\rho_t^n / (1+\phi\gamma_t) + \varkappa_t(\gamma_t, w_t) / \left(1+\phi\bar{\gamma}_t^h\right)}$$

where

$$\rho_t^b(\gamma_t, w_t) = \nu\left(1 - \varsigma_{bt}(\gamma_t, w_t)\, p_t^n\right)$$

The derivative of γ_{t+1} with respect to γ_t is

$$\frac{d\gamma_{t+1}}{d\gamma_t} = \frac{\partial \gamma_{t+1}}{\partial \gamma_t} + \frac{\partial \gamma_{t+1}}{\partial \varkappa_t}\frac{\partial \varkappa_t}{\partial \gamma_t} + \frac{\partial \gamma_{t+1}}{\partial \rho_t^b}\frac{\partial \rho_t^b}{\partial \varsigma_{bt}}\frac{\partial \varsigma_{bt}}{\partial \gamma_t}$$

and the derivative with respect to w_t is

$$\frac{d\gamma_{t+1}}{dw_t} = \frac{\partial \gamma_{t+1}}{\partial w_t} + \frac{\partial \gamma_{t+1}}{\partial \varkappa_t}\frac{\partial \varkappa_t}{\partial w_t} + \frac{\partial \gamma_{t+1}}{\partial \rho_t^b}\frac{\partial \rho_t^b}{\partial \varsigma_{bt}}\frac{\partial \varsigma_{bt}}{\partial w_t}$$

Let us consider each relevant term at a time. We have

$$\frac{\partial \gamma_{t+1}}{\partial \gamma_t} = \frac{\frac{\rho_t^b}{(1+\phi\gamma_t)^2}\left[\rho_t^n / (1+\phi\gamma_t) + \varkappa_t / \left(1+\phi\bar{\gamma}_t^h\right)\right] + \frac{\rho_t^n \phi}{(1+\phi\gamma_t)^2}\left[\rho_t^b \gamma_t / (1+\phi\gamma_t) + \varkappa_t \bar{\gamma}_t^h / \left(1+\phi\bar{\gamma}_t^h\right)\right]}{\left[\rho_t^n / (1+\phi\gamma_t) + \varkappa_t / \left(1+\phi\bar{\gamma}_t^h\right)\right]^2}$$

$$\frac{\partial \gamma_{t+1}}{\partial \varkappa_t} = \frac{(1+\phi\gamma_t)\left(1+\phi\bar{\gamma}_t^h\right)}{\left[\rho_t^n\left(1+\phi\bar{\gamma}_t^h\right) + \varkappa_t(1+\phi\gamma_t)\right]^2}\left(\bar{\gamma}_t^h \rho_t^n - \gamma_t \rho_t^b\right)$$

$$\frac{\partial \gamma_{t+1}}{\partial \rho_t^b} = \frac{1}{\rho_t^n / (1+\phi\gamma_t) + \varkappa_t / \left(1+\phi\bar{\gamma}_t^h\right)}\frac{\gamma_t}{1+\phi\gamma_t}$$

$$\frac{\partial \rho_t^b}{\partial \varsigma_{bt}} = -\nu p_t^n$$

$$\frac{\partial \gamma_{t+1}}{\partial w_t} = 0$$

Evaluating terms at steady state, using $\tilde{\rho}^b = \tilde{\rho}^n$ and $\tilde{\gamma} = \tilde{\gamma}^h$, we obtain

$$\frac{\partial \gamma'}{\partial \gamma} = \rho \quad \frac{\partial \gamma'}{\partial \varkappa} = 0 \quad \frac{\partial \gamma'}{\partial \rho^b} = \gamma \quad \frac{\partial \rho^b}{\partial \varsigma_b} = -\nu p^n$$

Substituting, we obtain

$$\frac{d\gamma'}{d\gamma} = \rho - \gamma \nu p^n \frac{\partial \varsigma_b}{\partial \gamma} \tag{S8}$$

$$\frac{d\gamma'}{dw} = -\gamma \nu p^n \frac{\partial \varsigma_b}{\partial w} \tag{S9}$$

C.2.5.6 Effect of composition and wages on search intensity

From the expressions in the previous sub-section, we need $\partial \varsigma_b / \partial \gamma$ and $\partial \varsigma_b / \partial w$. For any given composition γ_t and wage w_t, search intensity is given by

$$\begin{aligned} \varsigma_0 \varsigma_{bt} (\gamma_t, w_t)^{\eta_\varsigma} \quad = \quad & E_t \left\{ \Lambda_{t,t+1} p_t^n \left[\bar{H}_{t+1} \right.\right. \\ & \left.\left. -\lambda H_{t+1}^b (\gamma_{t+1}, w_t) - (1-\lambda) H_{t+1}^b (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1})) \right] \right\} \end{aligned}$$

We can write $\partial \varsigma_{bt} (\gamma_t, w_t) / \partial \gamma_t$ as follows

$$\begin{aligned} \frac{\partial \varsigma_{bt} (\gamma_t, w_t)}{\partial \gamma_t} \quad = \quad & - \left(\frac{\varsigma_{bt}^{1-\eta_\varsigma}}{\eta_\varsigma \varsigma_0} \right) E_t \left\{ \Lambda_{t,t+1} p_t^n \left[\lambda \frac{\partial H_{t+1}^b (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right.\right. \\ & + (1-\lambda) \frac{\partial H_{t+1}^b (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial \gamma_{t+1}} \\ & \left.\left. + (1-\lambda) \frac{\partial H_{t+1}^b (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial w_{t+1}^* (\gamma_{t+1})} \frac{\partial w_{t+1}^* (\gamma_{t+1})}{\partial \gamma_{t+1}} \right] \frac{d\gamma_{t+1}}{d\gamma_t} \right\} \end{aligned}$$

Letting

$$\tau \equiv \frac{\varsigma_b^{1-\eta_\varsigma} \beta p^n}{\eta_\varsigma \varsigma_0}$$

and evaluating at the steady state, we obtain

$$\frac{\partial \varsigma_b}{\partial \gamma} = -\tau \left(\frac{\partial H^b}{\partial \gamma} + (1-\lambda) \frac{\partial H^b}{\partial w} \frac{\partial w^* (\gamma)}{\partial \gamma} \right) \frac{d\gamma'}{d\gamma} \tag{S10}$$

We also have $\partial \varsigma_{bt}(\gamma_t, w_t)/\partial w_t$ as follows

$$\begin{aligned}
\frac{\partial \varsigma_{bt}}{\partial w_t} &= -\left(\frac{\varsigma_{bt}^{1-\eta_\varsigma}}{\eta_\varsigma \varsigma_0}\right) E_t\left\{\Lambda_{t,t+1} p_t^n \lambda \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_t)}{\partial w_t}\right\} \\
&\quad -\left(\frac{\varsigma_{bt}^{1-\eta_\varsigma}}{\eta_\varsigma \varsigma_0}\right) E_t\left\{\Lambda_{t,t+1} p_t^n \left[\lambda \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1-\lambda) \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*)}{\partial \gamma_{t+1}}\right.\right. \\
&\quad \left.\left. + (1-\lambda) \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*)}{\partial w_{t+1}^*} \frac{\partial w_{t+1}^*}{\partial \gamma_{t+1}}\right] \times \frac{d\gamma_{t+1}}{dw_t}\right\}
\end{aligned}$$

Evaluating at steady state gives

$$\frac{\partial \varsigma_b}{\partial w} = -\tau\lambda \frac{\partial H^b}{\partial w} - \tau\left(\frac{\partial H^b}{\partial \gamma} + (1-\lambda) \frac{\partial H^b}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma}\right) \frac{d\gamma'}{dw} \tag{S11}$$

C.2.5.7 Effect of composition and wages on worker surplus in bad matches

Finally, we derive expressions for $\partial H^b/\partial \gamma$ and $\partial H^b/\partial \gamma$. For any given composition γ_t and wage w_t, the worker surplus in a bad match is

$$\begin{aligned}
H_t^b(\gamma_t, w_t) &= \phi w_t - u_B - \nu c(\varsigma_{bt}) \\
&\quad + E_t\left\{\Lambda_{t,t+1}\left[\nu \varsigma_{bt} p_t^n \bar{H}_{t+1} - p_t \bar{H}_{t+1}^a\right]\right\} \\
&\quad + \nu(1-\varsigma_{bt} p_t^n) E_t\left\{\Lambda_{t,t+1}\left[\lambda H_{t+1}^b(\gamma_{t+1}, w_t)\right.\right. \\
&\quad \left.\left. + (1-\lambda) H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))\right]\right\}
\end{aligned}$$

We can write $\partial H_t^b(\gamma_t, w_t)/\partial \gamma_t$ as follows

$$\begin{aligned}
\frac{\partial H_t^b(\gamma_t, w_t)}{\partial \gamma_t} &= \nu(1-\varsigma_{bt} p_t^n) E_t\left\{\Lambda_{t,t+1}\left[\lambda \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}}\right.\right. \\
&\quad + (1-\lambda) \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial \gamma_{t+1}} \\
&\quad \left.\left. + (1-\lambda) \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial \gamma_{t+1}}\right] \times \frac{d\gamma_{t+1}}{d\gamma_t}\right\}
\end{aligned}$$

where $H_t^b(\gamma_t, w_t)$ is maximized with respect to $\varsigma_{bt}(\gamma_t, w_t)$, so that in taking the derivative with respect to γ_t, we can hold $\varsigma_{bt}(\gamma_t, w_t)$ fixed at its optimal value.

Evaluating at steady state gives

$$\frac{\partial H^b}{\partial \gamma}\left(1 - \rho\beta \frac{d\gamma'}{d\gamma}\right) = \rho\beta(1-\lambda) \frac{\partial H^b}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{d\gamma'}{d\gamma} \tag{S12}$$

We then have $\partial H_t^b(\gamma_t, w_t)/\partial w_t$ as

$$\begin{aligned}
\frac{\partial H_t^b(\gamma_t, w_t)}{\partial w_t} &= \phi + \nu(1 - \varsigma_{bt} p_t^n) E_t \left\{ \Lambda_{t,t+1} \lambda \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_t)}{\partial w_t} \right\} \\
&+ \nu(1 - \varsigma_{bt} p_t^n) E_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right. \right. \\
&+ (1-\lambda) \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial \gamma_{t+1}} \\
&\left. \left. + (1-\lambda) \frac{\partial H_{t+1}^b(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial \gamma_{t+1}} \right] \times \frac{d\gamma_{t+1}}{dw_t} \right\}
\end{aligned}$$

where we have used the envelope condition.

Evaluating at steady state gives the last equation of the system

$$\frac{\partial H^b}{\partial w} \left(1 - \rho\beta\lambda - \rho\beta(1-\lambda) \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{d\gamma'}{dw} \right) = \phi + \rho\beta \frac{\partial H^b}{\partial \gamma} \frac{d\gamma'}{dw} \tag{S13}$$

C.2.5.8 System and solution We have a system of 13 equations, given by equations (S1)-(S13), in 13 unknowns, given by

$$\left\{ \frac{\partial w^*(\gamma)}{\partial \gamma}, \frac{\partial H}{\partial \gamma}, \frac{\partial H}{\partial w}, \frac{\partial J}{\partial \gamma}, \frac{\partial J}{\partial w}, \frac{d\rho}{d\gamma}, \frac{d\rho}{dw}, \frac{d\gamma'}{d\gamma}, \frac{\partial \gamma'}{\partial w}, \frac{\partial \varsigma_b}{\partial \gamma}, \frac{\partial \varsigma_b}{\partial w}, \frac{\partial H^b}{\partial \gamma}, \frac{\partial H^b}{\partial w} \right\}$$

It is easy to see that $\partial w^*(\gamma)/\partial \gamma = 0$ is a solution to the system. That is, in the steady state, the outcome of the current wage bargain is independent of current composition, as long as workers and firms believe that the same is true of the outcome of future wage bargains. The full solution to the system is as follows:

$$\begin{cases} \frac{\partial w^*(\gamma)}{\partial \gamma} = \frac{\partial H}{\partial \gamma} = \frac{\partial J}{\partial \gamma} = \frac{d\rho}{d\gamma} = \frac{\partial \varsigma_b}{\partial \gamma} = \frac{\partial H^b}{\partial \gamma} = 0 \\ \frac{\partial H}{\partial w} = \frac{1}{1-\rho\beta\lambda} \\ \frac{\partial J}{\partial w} - \frac{1-\frac{\partial \rho}{\partial w}\beta J}{1-\lambda\beta} \\ \frac{d\rho}{dw} = -\frac{\phi\gamma}{1+\phi\gamma}\nu p^n \frac{\partial \varsigma_b}{\partial w} \\ \frac{d\gamma'}{d\gamma} = \rho \\ \frac{d\gamma'}{dw} = -\gamma\nu p^n \frac{\partial \varsigma_b}{\partial w} \\ \frac{\partial \varsigma_b}{\partial w} = -\frac{\tau\lambda\phi}{1-\rho\beta\lambda} \\ \frac{\partial H^b}{\partial w} = \frac{\phi}{1-\rho\beta\lambda} \end{cases}$$

C.2.6 Derivation of surplus approximations

We now prove the first-order approximations for the period-ahead surpluses at renegotiating firms stated in equations (42), (43) and (44) in the first sub-section. To do that, we will use the steady state results just proved.

Specifically, we derive the approximation in equation (42), given by

$$\begin{aligned} \widehat{H}_{t+1}\left(\gamma_{t+1}, w^*_{t+1}\left(\gamma_{t+1}\right)\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1}, \bar{w}^*_{t+1}\right) \\ \widehat{H}_{t+1}\left(\gamma_{t+1}, \bar{w}^*_{t}\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1}, \bar{w}^*_{t+1}\right) + \eta_{Hw}\left(\widehat{\bar{w}}^*_t - \widehat{\bar{w}}^*_{t+1}\right) \end{aligned}$$

Similar derivations apply to equations (43) and (44).

Consider the worker surplus in good matches, H. It is a function of the two firm-specific states, composition and wage, and the aggregate state. Note that the aggregate state not appear explicitly as an argument of H as it is captured by our notation with the time index on the function H.

Denoting the aggregate state at time t with $\mathbf{s}_t$, we can then loglinearize the worker surplus at time t for any wage, w_t, and any composition, γ_t, as follows:

$$\widehat{H}_t\left(\gamma_t, w_t\right) = \eta_{H\gamma}\widehat{\gamma}_t + \eta_{Hw}\widehat{w}_t + \eta_{Hs}\widehat{\mathbf{s}}_t \tag{45}$$

where $\eta_{H\gamma}$, η_{Hw}, and η_{Hs} are the steady state elasticities of H with respect to composition, wage and aggregate state.

Using the approximation in (45), we can then write:

$$\begin{aligned}\widehat{H}_{t+1}\left(\gamma_{t+1},w^*_{t+1}\left(\gamma_{t+1}\right)\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1},\bar{w}^*_{t+1}\right)+\eta_{H\gamma}\left(\widehat{\gamma}_{t+1}-\widehat{\bar{\gamma}}_{t+1}\right)+\eta_{Hw}\left(\widehat{w}^*_{t+1}\left(\gamma_{t+1}\right)-\widehat{\bar{w}}^*_{t+1}\right)\\ \widehat{H}_{t+1}\left(\gamma_{t+1},\bar{w}^*_t\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1},\bar{w}^*_{t+1}\right)+\eta_{H\gamma}\left(\widehat{\gamma}_{t+1}-\widehat{\bar{\gamma}}_{t+1}\right)+\eta_{Hw}\left(\widehat{\bar{w}}^*_t-\widehat{\bar{w}}^*_{t+1}\right)\end{aligned} \tag{46}$$

where the terms in the aggregate state $\widehat{\mathbf{s}}_{t+1}$ cancel out.

We also have the following approximation:

$$\widehat{w}^*_{t+1}\left(\gamma_{t+1}\right)-\widehat{\bar{w}}^*_{t+1}=\eta_{w\gamma}\left(\widehat{\gamma}_{t+1}-\widehat{\bar{\gamma}}_{t+1}\right) \tag{47}$$

where $\eta_{w\gamma}$ is the steady state elasticity of the contract wage with respect to composition and terms in the aggregate state $\widehat{\mathbf{s}}_{t+1}$ cancel out.

Substituting (47) in the first line of (46) we obtain:

$$\begin{aligned}\widehat{H}_{t+1}\left(\gamma_{t+1},w^*_{t+1}\left(\gamma_{t+1}\right)\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1},\bar{w}^*_{t+1}\right)+\eta_{H\gamma}\left(\widehat{\gamma}_{t+1}-\widehat{\bar{\gamma}}_{t+1}\right)+\eta_{Hw}\eta_{w\gamma}\left(\widehat{\gamma}_{t+1}-\widehat{\bar{\gamma}}_{t+1}\right)\\ \widehat{H}_{t+1}\left(\gamma_{t+1},\bar{w}^*_t\right) &= \widehat{H}_{t+1}\left(\bar{\gamma}_{t+1},\bar{w}^*_{t+1}\right)+\eta_{H\gamma}\left(\widehat{\gamma}_{t+1}-\widehat{\bar{\gamma}}_{t+1}\right)+\eta_{Hw}\left(\widehat{\bar{w}}^*_t-\widehat{\bar{w}}^*_{t+1}\right)\end{aligned} \tag{48}$$

The final step to derive equation (42) is to use $\eta_{H\gamma}=\eta_{w\gamma}=0$, in turn resulting from $\partial H/\partial\gamma=\partial w^*/\partial\gamma=0$ in the steady state, a result we have just proved. Substituting $\eta_{H\gamma}=\eta_{w\gamma}=0$ in (48), we finally obtain equation (42).

Equations (43) and (44) are obtained with similar steps, using in this case $\eta_{J\gamma}=\eta_{H^b\gamma}=\eta_{w\gamma}=0$, in turn due to $\partial J/\partial\gamma=\partial H^b/\partial\gamma=\partial w^*/\partial\gamma=0$ in the steady state.

C.3 Wage growth of job changers

In this section, we derive expressions for the flow shares of the various types of job-to-job flows; and we derive an expression for the average wage growth of job changers.

C.3.1 Job-to-job flows

The model includes two types of job-to-job movers: those who search with variable search intensity from bad matches and those in good matches who are forced to search for non economic reasons, i.e., who are subject to a reallocation shock. Since workers in bad matches searching on the job only accept good matches, the first type of job changers leads only to bad-to-good flows. The second type of job changers instead leads to both good-to-bad and good-to-good flows. We have the following job-to-job flows:

$$\begin{aligned}\text{Bad to good :}\quad & \nu\bar{\varsigma}_{bt}\xi p_t\bar{b}_t\\ \text{Good to bad :}\quad & \nu\varsigma_n\left(1-\xi\right)p_t\bar{n}_t\\ \text{Good to good :}\quad & \nu\varsigma_n\xi p_t\bar{n}_t\end{aligned}$$

Summing over the flows we obtain total job flows as:

$$\nu\left(\bar{\varsigma}_{bt}\xi\bar{b}_t+\varsigma_n\bar{n}_t\right)p_t$$

The shares of flows over total flows then are defined as:

$$\begin{aligned}\delta_{BG,t} &= \frac{\bar{\varsigma}_{bt}\xi\bar{\gamma}_t}{\bar{\varsigma}_{bt}\xi\bar{\gamma}_t+\varsigma_n}\\ \delta_{GB,t} &= \frac{\varsigma_n\left(1-\xi\right)}{\bar{\varsigma}_{bt}\xi\bar{\gamma}_t+\varsigma_n}\\ \delta_{GG,t} &= \frac{\varsigma_n\xi}{\bar{\varsigma}_{bt}\xi\bar{\gamma}_t+\varsigma_n}\end{aligned}$$

C.3.2 Average wage growth of job changers

Let $\bar{g}_t^w$ denote the average wage growth of continuing workers and $\bar{g}_t^{EE}$ the average wage growth of workers making and employment-to-employment transition.

Up to a first order, $\bar{g}_t^{EE}$ can be written as:

$$\bar{g}_t^{EE}=\delta_{BG,t-1}\log\left(\frac{\bar{w}_t}{\phi\bar{w}_{t-1}}\right)+\delta_{GB,t-1}\log\left(\frac{\phi\bar{w}_t}{\bar{w}_{t-1}}\right)+\delta_{GG,t-1}\log\left(\frac{\bar{w}_t}{\bar{w}_{t-1}}\right)$$

Simplifying, we obtain:

$$\bar{g}_t^{EE}=\bar{g}_t^w+\Delta\bar{\alpha}_t^{EE}$$

with

$$\bar{g}_t^w=\log\left(\frac{\bar{w}_t}{\bar{w}_{t-1}}\right)$$

and

$$\Delta\bar{\alpha}_t^{EE}=\left(-\log\phi\right)\left(\delta_{BG,t-1}-\delta_{GB,t-1}\right)$$

Thus, average wage growth of new hires that are job changers equals average wage growth of continuing workers plus a composition component measuring the change in match quality among job changers. The composition component equals 0 if match quality is homogeneous ($\phi=1$).

Loglinearizing the average gross wage growth of job changers, we obtain:

$$\bar{g}_t^{EE} = \tilde{g}^{EE} + \frac{1}{1+\Delta\tilde{\alpha}^{EE}}\bar{g}_t^{w} + \frac{\Delta\tilde{\alpha}^{EE}}{1+\Delta\tilde{\alpha}^{EE}}\Delta\bar{\alpha}_t^{EE}$$

Loglinearizing the compositional effect, we obtain:

$$\widehat{\Delta\bar{\alpha}}_t^{EE} = \frac{1}{\tilde{\delta}_{BG}-\tilde{\delta}_{GB}}\left(\tilde{\delta}_{BG}\widehat{\delta}_{BG,t-1} - \tilde{\delta}_{GB}\widehat{\delta}_{GB,t-1}\right)$$

with

$$\hat{\delta}_{BG,t} = \frac{1}{1+\tilde{\gamma}}\widehat{\bar{\gamma}}_t + \left(1-\tilde{\delta}_{BG}\right)\widehat{\bar{\varsigma}}_{bt}$$

$$\hat{\delta}_{GB,t} = -\frac{\tilde{\gamma}}{1+\tilde{\gamma}}\widehat{\bar{\gamma}}_t - \tilde{\delta}_{BG}\widehat{\bar{\varsigma}}_{bt}$$

Rearranging, we find the expression relating the composition effect to variable search intensity of workers in bad matches and firm average composition:

$$\widehat{\Delta\bar{\alpha}}_t^{EE} = \frac{\tilde{\delta}_{BG}+\tilde{\gamma}\tilde{\delta}_{GB}}{(1+\tilde{\gamma})\left(\tilde{\delta}_{BG}-\tilde{\delta}_{GB}\right)}\widehat{\bar{\gamma}}_{t-1} + \frac{1-\left(\tilde{\delta}_{BG}-\tilde{\delta}_{GB}\right)}{\left(\tilde{\delta}_{BG}-\tilde{\delta}_{GB}\right)}\tilde{\delta}_{BG}\widehat{\bar{\varsigma}}_{bt-1}$$

C.4 Steady state

Here we describe the steady state for the labor market, conditional on the steady-state marginal product of labor, $\tilde{a}$, which is determined as in the conventional neoclassical growth model and is independent of the labor market equilibrium. The key labor market variables are the hiring rate, $\tilde{\varkappa}$, the search intensity of workers in bad matches, $\tilde{\varsigma}_b$, the wage, $\tilde{w}$, employment in good and bad matches, $\tilde{n}$ and $\tilde{b}$, the retention rates for good and bad matches, $\tilde{\rho}^n$ and $\tilde{\rho}^b$, the retention rate per unit of labor quality, $\tilde{\rho}$, the job-finding probability, $\tilde{p}$, searchers, $\tilde{s}$, unemployment, $\tilde{u}$, composition, $\tilde{\gamma}$, and vacancies, $\tilde{v}$. Further, relevant to the composition effect described in the previous section, are the average wage growth of job changers, $\tilde{g}^{EE}$, the average flow share of bad-to-good transitions, $\tilde{\delta}_{BG}$, and the average flow share of good-to-bad transitions, $\tilde{\delta}_{GB}$.

Equations (49)-(58) below determine $\tilde{\varkappa}$, $\tilde{\varsigma}_b$, $\tilde{w}$, $\tilde{n}$, $\tilde{b}$, $\tilde{\rho}^n$, $\tilde{\rho}^b$, $\tilde{\rho}$, $\tilde{p}$, and $\tilde{s}$. First, there are the three key behavioral relations: the hiring condition, the search intensity condition and the condition for the wage bargain:

$$\kappa\tilde{\varkappa} = \beta\tilde{J} \tag{49}$$

$$\varsigma_0\tilde{\varsigma}_b^{\eta_\varsigma} = \beta\tilde{p}\xi\left(\tilde{H}-\tilde{H}^b\right) \tag{50}$$

$$\eta\tilde{H} = (1-\eta)\tilde{J} \tag{51}$$

where the firm and worker surpluses are given by

$$\tilde{J} = \tilde{a} - \tilde{w} + \frac{\kappa}{2}\tilde{\varkappa}^2 + \tilde{\rho}\beta\tilde{J}$$

$$\tilde{H} = \tilde{w} - u_B - \nu\varsigma_0 \frac{\varsigma_n^{1+\eta_\varsigma}}{1+\eta_\varsigma} + \beta\left[\tilde{\rho}^n\tilde{H} - \tilde{p}\tilde{H}^a + \left(\nu - \tilde{\rho}^n\right)\tilde{H}^a\right]$$

$$\tilde{H}^b = \phi\tilde{w} - u_B - \nu\varsigma_0 \frac{\tilde{\varsigma}_b^{1+\eta_\varsigma}}{1+\eta_\varsigma} + \beta\left[\tilde{\rho}^b\tilde{H}^b - \tilde{p}\tilde{H}^a + \left(\nu - \tilde{\rho}^b\right)\tilde{H}\right]$$

with

$$\tilde{H}^a = \xi\tilde{H} + (1-\xi)\tilde{H}^b$$

Second, in the steady state, hiring equals separations for both good and bad matches:

$$\tilde{p}\xi\tilde{s} = (1 - \tilde{\rho}^n)\,\tilde{n} \tag{52}$$

$$\tilde{p}\,(1-\xi)\left(\tilde{s} - \nu\tilde{\varsigma}_b\tilde{b}\right) = \left(1 - \tilde{\rho}^b\right)\tilde{b} \tag{53}$$

where the survival rates are given by

$$\tilde{\rho}^n = \nu\,(1 - \varsigma_n\tilde{p}) \tag{54}$$

$$\tilde{\rho}^b = \nu\,(1 - \tilde{\varsigma}_b\tilde{p}\xi) \tag{55}$$

and searchers are given by

$$\tilde{s} = \left(1 - \tilde{n} - \tilde{b}\right) + \nu\tilde{\varsigma}_b\tilde{b} + \nu\varsigma_n\tilde{n} \tag{56}$$

Third, since firms employ labor quality units by hiring workers in both good and bad matches, in the steady state the hiring rate in units of labor quality equals the separation rate per unit of labor quality:

$$\tilde{\varkappa} = 1 - \tilde{\rho} \tag{57}$$

where

$$\tilde{\rho} = \frac{\tilde{\rho}^n\tilde{n} + \phi\tilde{\rho}^b\tilde{b}}{\tilde{n} + \phi\tilde{b}} \tag{58}$$

Unemployment, $\tilde{u}$, composition, $\tilde{\gamma}$, and vacancies, $\tilde{v}$, are then pinned down by equations (59)-(61), given by:

$$1 = \tilde{u} + \tilde{n} + \tilde{b} \tag{59}$$

$$\tilde{\gamma} = \frac{\tilde{b}}{\tilde{n}} \tag{60}$$

$$\tilde{p}\tilde{s} = \sigma_m \tilde{s}^{\sigma} \tilde{v}^{1-\sigma} \tag{61}$$

Finally, $\tilde{g}^{EE}$, $\tilde{\delta}_{BG}$, and $\tilde{\delta}_{GB}$ are determined by equations (62)-(64) as follows:

$$\tilde{g}^{EE} = -\log \phi \left(\tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right) \tag{62}$$

$$\tilde{\delta}_{BG} = \frac{\tilde{\gamma} \tilde{\varsigma}_b \xi}{\tilde{\gamma} \tilde{\varsigma}_b \xi + \varsigma_n} \tag{63}$$

$$\tilde{\delta}_{GB} = \frac{\varsigma_n (1 - \xi)}{\tilde{\gamma} \tilde{\varsigma}_b \xi + \varsigma_n} \tag{64}$$

C.5 Calibration strategy

In this section we discuss how our calibration strategy leads to identification of the key model parameters, in particular those driving the composition effect. The steady state for the labor market described in the previous section includes 12 parameters: β, σ_m, σ, η, ν, ς_n, ϕ, ξ, u_B, ς_0, η_ς, κ. As we discuss in the main text, the first 4 parameters are either normalized or calibrated using external sources. The remaining 8 parameters are calibrated to target model-relevant moments: the steady state wage change of workers making a direct job-to-job transition, $\tilde{g}^{EE}$; the steady state value of the share of bad-to-good flows out of total job flows, $\tilde{\delta}_{BG}$, and its cyclicality, $\eta_{\delta_{BG,t},u_t}$; the steady state probabilities of making an unemployment to employment transition, p^{UE}, an employment to unemployment transition, p^{EU}, and an employment to employment transition, p^{EE}; and the relative value of non-work to work, $\bar{u}_T$. Finally, as we discussed in the main text, we impose a steady state restriction, whereby retention rates in good and bad matches are equal in the steady state, $\tilde{\rho}^n = \tilde{\rho}^b$.

The calibration strategy leads to a unique solution for the steady state values and the internally calibrated parameters. Here we show how the key parameter values relate to the targets.

First, the separation rate into unemployment, $1 - \nu$, is simply given by:

$$1 - \nu = p^{EU}$$

Second, to explain how ξ is determined, we start by noting that, given the steady state restriction, the average flow share of bad-to-good only depends on the relative number of workers in bad and good matches, as follows:

$$\tilde{\delta}_{BG} = \frac{\tilde{\gamma}}{1 + \tilde{\gamma}}$$

This pins down $\tilde{\gamma}$, given a target for $\tilde{\delta}_{BG}$. Combining then equations (52)-(56) and (58)-(60

), we derive a relation between ξ, $\tilde{\gamma}$, and the targets, p^{EE} and p^{EU}, given by:

$$\tilde{\gamma} = \frac{(1-\xi)\left(p^{EE}+p^{EU}\right)}{p^{EE}+\xi p^{EU}}$$

This relation uniquely pins down ξ. The inverse productivity parameter, ϕ, is then computed using the expression for the average wage growth of job changers:

$$\tilde{g}^{EE} = -\log\phi\left(\tilde{\delta}_{BG}-\tilde{\delta}_{GB}\right)$$

given the target for $\tilde{g}^{EE}$, the target for $\tilde{\delta}_{BG}$ and the average good-to-bad flow share $\tilde{\delta}_{GB}$, in turn computed as:

$$\tilde{\delta}_{GB} = \frac{1-\xi}{1+\tilde{\gamma}}$$

The parameter ς_n is pinned down, for given targets for the transition probabilities, by the relation:

$$p^{EE} = \left(1-p^{EU}\right)p^{UE}\varsigma_n$$

where the probability that a worker in a good match makes an job-to-job transition, p^{EE}, equals the probability of surviving within the match, $\left(1-p^{EU}\right)$, times the job finding rate, p^{UE}, times the search intensity, ς_n. The parameters u_B, ς_0 and κ are computed solving a subset of the steady state system, equations (49)-(51), given a target for $\bar{u}_T$ and a value for η_ς (and other computed parameters and steady state values). Finally, the key parameter η_ς, driving the elasticity of search intensity for workers in bad matches to the gain of making a bad-to-good transition, is set to implement the target η_{BG}.

C.6 The measured user cost of labor

In this section, we derive the loglinear version of the measured user cost of labor, ucl_t^m, introduced in Section 6, and given by:

$$ucl_t^m = w_{t,t}^m + E_t\left\{\sum_{s=1}^{\infty}(\beta\rho^m)^s\left(w_{t,t+s}^m - w_{t+1,t+s}^m\right)\right\}, \tag{65}$$

where $w_{t,t+s}^m$ denotes the average measured wage of workers at $t+s$ who still occupy the job that they were hired into at time t and ρ^m the average measured retention rate, equal in our model to $\tilde{\rho}$.

Let γ_{t-1}^h denote the composition of new hires in t and $\gamma_{t-1,t+s}^h$ the composition of workers at $t+s$ who still occupy the job they were hired into at time t.[6] The latter can be expressed

[6] Recall workers are hired at the end of the period and begin as new hires the subsequent period. Hence, the ratio of bad-to-good matches among new hires at time t is given by γ_{t-1}^h.

recursively as follows:

$$\gamma^h_{t-1,t+s} = \begin{cases} \gamma^h_{t-1} & \text{if } s = 0 \\ \gamma^h_{t-1,t+s} \dfrac{\rho^b_{t+s}}{\rho^n_{t+s}} & \text{if } s > 0 \end{cases}, \tag{66}$$

where ρ^b_{t+s} and ρ^n_{t+s} are the retention rates at time $t+s$ of workers employed in bad and good matches.

Then, $w^m_{t,t+s}$ can be written as:

$$w^m_{t,t+s} = \frac{1+\phi\gamma^h_{t-1,t+s}}{1+\gamma^h_{t-1,t+s}} w_{t+s}, \tag{67}$$

where w_{t+s} is the contract wage at time $t+s$.

Substituting (67) in (65), loglinearizing and simplifying yields:

$$\widehat{ucl}^m_t = \widehat{w}_t + \widehat{c}^{ucl}_t, \tag{68}$$

where the measure user cost, $\widehat{ucl}^m_t$, is the sum of two terms: the true user cost of labor, $\widehat{w}_t$, and a compositional component, $\widehat{c}^{ucl}_t$, in turn given by:

$$\widehat{c}^{ucl}_t = -\Psi\left(\widehat{\gamma}^h_{t-1} + \frac{\tilde{\rho}\beta}{1-\tilde{\rho}\beta}\left[\left(\widehat{\rho}^b_t - \widehat{\rho}^n_t\right) + \left(\widehat{\gamma}^h_t - \widehat{\gamma}^h_{t-1}\right)\right]\right), \tag{69}$$

with

$$\Psi = \frac{(1-\phi)\,\tilde{\gamma}^h}{\left(1+\tilde{\gamma}^h\right)\left(1+\phi\tilde{\gamma}^h\right)}. \tag{70}$$

C.7 Lateral movements

A key maintained hypothesis in our analysis is that workers and firms can expect that workers searching on-the-job will not want to voluntarily make lateral movements, that is, to voluntarily move from a job of a given quality to another job of the same quality. As we discuss shortly, this hypothesis simplifies how workers employed in bad matches form expectations when choosing search intensity. At the same time, it also justifies our ruling out in the model of variable search intensity by workers in good matches (since it eliminates any motive for moving to improve the pecuniary gain).

Here we demonstrate that this condition holds to a reasonable approximation. Put differently, under our parameterization, expected gains from making a lateral movement are negligible. As a consequence, the introduction of a small moving cost would suffice to rule them out. Intuitively, gains from moving to a same-quality job can only come from temporary dispersion in wages associated to infrequent bargaining. In particular, in presence of a small moving cost, workers can expect to be willing to make a lateral movement only if their wages are (i) substantially below the average and (ii) are not likely to be renegotiated for sometime. However, the likelihood a worker is in this situation in our model is of trivial quantitative importance, due to the transitory nature on average of wage differentials due to staggered contracting.[7]

To start, consider a general framework where lateral movements are allowed for. Let $V_{t+1}^{s,i}$ be the value of on-the-job search at $t+1$, conditional on finding a job at t, for $i = n, b$.

For a worker employed in bad match, the expected value of on-the-job search, conditional on a match, can be written as the sum of three terms:

$$E_t \left\{ V_{t+1}^{s,b} \right\} = E_t \left\{ V_{t+1}^{b} \right\} + \xi E_t \left\{ \bar{V}_{t+1}^{n} - V_{t+1}^{b} \right\} + (1-\xi) L_t^b \tag{71}$$

The first term is the expected value of continuing with the same job, $E_t \left\{ V_{t+1}^{b} \right\}$; the second term is the expected gain if the new match is good, in which case the worker makes a bad-to-good move; the third term is the expected gain from moving to another bad match, i.e., the expected gain from a lateral move, with L_t^b to be defined shortly. The problem faced by workers in bad matches optimally choosing search intensity that we formulate in the main text corresponds to setting $L_t^b = 0$. This greatly simplifies the solution to the search intensity problem.

For a worker employed in a good match, the value of on-the-job search is the sum of only two terms (since the worker will not voluntarily move to a bad match):

$$E_t \left\{ V_{t+1}^{s,n} \right\} = E_t \left\{ V_{t+1}^{n} \right\} + \xi L_t^n \tag{72}$$

The first term is the expected value of continuing with the same job, $E_t \left\{ V_{t+1}^{n} \right\}$; the second term is the expected gain from moving to another good match, that is, the expected gain from a lateral move. Clearly, with $L_t^n = 0$, workers in good matches have no incentive to search on-the-job as they cannot improve on their current status. Accordingly, we rule out an optimal choice for variable search intensity for workers in good matches.

We now demonstrate that $L_t^i = 0$, for $i = n, b$, holds to a reasonable approximation.

Consider the lateral movement term, L_t^i, that is, the expected gain from making a lateral movement, conditional on finding a job of the same quality, for $i = n, b$. This can be written

[7] Ranking firms by contract wage, our model implies a period 0.07 percent wage gain for workers making a lateral movement from 10$^{\text{th}}$ to 90$^{\text{th}}$ percentile firm. This calculation does not account for the transitory nature of wage gains from lateral movements, implying a far lower permanent wage improvement associated with such a job transition relative to a transition with improvement in match quality. We discuss this in more detail below.

as

$$L_t^i = E_t \left\{ \int_{\gamma,w} \max\left\{ V_{t+1}^i\left(\gamma', w'\right) - V_{t+1}^i, 0\right\} dF_t\left(\gamma, w\right) \right\} \tag{73}$$

where $dF_t(\gamma, w) \equiv \frac{\varkappa_t(\gamma,w)}{\bar{\varkappa}_t} dG_t(\gamma, w)$ and where the expression takes into account that a worker will make a lateral move only if the value at the new match, $V_{t+1}^i(\gamma', w')$, exceeds the value at the current match, V_{t+1}^i.

Through an involved series of derivations[8], it can be shown that a first-order approximation of the lateral movement term around the steady state is given by

$$L_t^i = \lambda \frac{\partial \tilde{V}^i}{\partial \tilde{w}} \int_{w > \tilde{w}} \left(w - \tilde{w}\right) dF_t\left(\gamma, w\right) \tag{74}$$

where $\tilde{w}$ is the steady state wage and $\partial \tilde{V}^i / \partial \tilde{w}$ is the steady state gain from a higher wage. Intuitively, up to a first order, L_t^i equals the average gain at time t of moving from a match of a given quality to a match of the same quality but higher wage than the average, given the distribution of wages (per unit of labor quality) at time t. The wage dispersion term, $\int_{w>\tilde{w}} (w - \tilde{w})\, dF_t(\gamma, w)$, is weighted by the likelihood next period that the wage at the new match is not renegotiated, λ, and by the steady state lifetime utility gain, $\partial \tilde{V}^i / \partial \tilde{w}$.[9]

Using expression (74), we then compute the lateral movement term. The difficult object to construct is a counterpart for $dF_t(\gamma, w)$. We proceed as follows. First, we simulate a time series for the average contract wage from the model. Second, we pick an integer T large enough so that the probability that a firm has not renegotiated its wage for T period is approximately 0. We treat T as a truncation point, that is, we assume that the probability at t that $w_t = w_{t-T-h}^*$ for $h > 0$ is zero. Once the truncation point is determined, the probability that a firm has a wage $w_t = w_{t-h}^*$ is given by[10]

$$\Pr\left(w_t = w_{t-h}^*\right) = \begin{cases} \lambda^h \left(1 - \lambda\right) & \text{if } h < T \\ \lambda^h & \text{if } h = T \end{cases} \tag{75}$$

After computing the approximate wage distribution, we calculate L_t^i as follows

$$\lambda \frac{\partial \tilde{V}^i}{\partial \tilde{w}} \left[\sum_{h=0}^{T} \max\left\{ w_{t-h}^* - \tilde{w}, 0 \right\} \Pr\left(w_t = w_{t-h}^*\right) \right] \tag{76}$$

[8] The derivations are available upon request.

[9] The utility gain $\partial \tilde{V}^i / \partial \tilde{w}$ equals $\phi / (1 - \rho\beta\lambda)$ for $i = b$ and $1/(1 - \rho\beta\lambda)$ for $i = n$ and accounts for the expected duration of a match×contract.

[10] We note that in constructing the relevant distribution, we ignore the fact that firm hiring rates will vary in the cross section with wages. However, given that (i) hiring rates are decreasing in the wage and (ii) the gain from a lateral movement is increasing in the wage at the new firm, we are constructing an upper bound for the average gain from a lateral move.

We then take the average over time and express it as a percentage of the steady state lifetime value V^i. We obtain the following result: the expected percent gain from a lateral movement is tiny. Precisely, it equals 0.032 percent for workers in bad matches and 0.040 percent for workers in good matches.

Importantly, wage dispersion generated by staggered bargaining is negligible not only in absolute terms, but also relative to the dispersion generated by differences in match quality. To see this, we compare the expected percent wage gain from an improvement in match quality, given by $(1-\phi)/\phi$, to the expected percent wage gain from a lateral movement, given by

$$\left(\frac{1-\beta\lambda}{1-\rho\beta\lambda}\right)\lambda\int_{w>\tilde{w}}\left(\frac{w-\tilde{w}}{\tilde{w}}\right)dF_t\left(\gamma,w\right) \tag{77}$$

where the latter is corrected by an adjustment term, $(1-\beta\lambda)/(1-\rho\beta\lambda)$, capturing the fact that while gains from lateral movements last over the employment duration as long as the wage is not recontracted, gains from improved match quality last over the entire employment duration. Taking averages over time, the first equals 0.2836, while the second equals 0.0100.

Finally, we note that while we assume that workers searching on-the-job from bad matches expect they will not want to make a lateral move, they will still run into other matches of the same quality but with higher wages. We similarly rule out the possibility that they move to those new matches. To address this point, we compute measures of the distribution of ex-post potential wage gains at time t. Given random search, the probability that a worker with wage $w = w^*_{t-h}$ finds a firm with wage $w' = w^*_{t-j}$ equals $\Pr(w = w^*_{t-h}) \cdot \Pr(w' = w^*_{t-j})$. We compute the 75th, 90th, 95th and 99th percentiles wage gains from lateral moves at each t, expressed as a percent wage gain corrected as above by the adjustment term, and take averages over time. We obtain the following numbers: the 75th percentile is 0.0051; the 90th percentile is 0.0110; the 95th percentile is 0.0145 and the 99th percentile is 0.0196. Compared to the gains from moving to a better quality match, these are small numbers.

C.8 Transition function

We now define the law of motion for the distribution function, G_t. Let C and W be the sets of possible compositions and wages. Define the Cartesian product of the worker/firm state space to be $S \equiv C \times W$ with σ-algebra Σ with typical subset $\mathcal{S} = (\mathcal{C} \times \mathcal{W})$. Define the transition function $Q_{\mathbf{s},\,\mathbf{s}'}\left((\gamma,w),\mathcal{C}\times\mathcal{W}\right)$ as the probability that an individual retained or hired by a firm characterized by (γ, w) transits to the set $\mathcal{C} \times \mathcal{W}$ next period when the aggregate state transits from $\mathbf{s}$ to $\mathbf{s}'$. Then $Q_{\mathbf{s},\mathbf{s}'}$ satisfies

$$
\begin{aligned}
Q_{t,t+1}\left((\gamma_t, w_t), \mathcal{C} \times \mathcal{W}\right) = {} & I(\gamma_{t+1}(\gamma_t, w_t) \in \mathcal{C}) \\
& \times \Bigg[(1-\lambda) I\left(w_t^*\left(\gamma_{t+1}(\gamma_t, w_t)\right) \in \mathcal{W}\right) \frac{\varkappa_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\bar{\varkappa}_t + \bar{\rho}_t} \\
& + \lambda I\left(w_t \in W\right) \frac{\varkappa_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\bar{\varkappa}_t + \bar{\rho}_t}\Bigg]
\end{aligned}
$$

where $I(\cdot)$ is the indicator function. Then,

$$
G_{t+1}(\mathcal{C} \times \mathcal{W}) = \int_{(\gamma_t, w_t) \in \mathcal{C} \times W} Q_{t,t+1}\big((\gamma_t, w_t), \mathcal{C} \times \mathcal{W})\big) dG_t(\gamma_t, w_t).
$$